Stochastic Programming Model in Agricultural Production Decision Support System

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Abstract. *Agricultural production depends on various factors which are frequently impossible to control to a sufficient degree. Consequently, it is not possible to determine precisely parameter values, which is a major problem for decision-makers when putting up a plan for the type and quantity of agricultural output that should be produced and offered on the market. Since applying deterministic models cannot yield satisfactory results in such circumstances, it is necessary to improve the decision support system by implementing stochastic programming. Its basic feature is defining parameters of the model as random variables. In order to establish solutions to the problem set in this way, the stochastic programming model has to be transformed into a deterministic form, which then results in one or more constraints of non-linear character. Given that this form of mathematical optimization is very complex to solve, it requires managers to use computers and adequate software in order to carry it out. In this paper, the proposed model of decision-making in agricultural production, based on computer optimization, is first considered from a theoretical viewpoint. After that, the procedure of its construction and solution is illustrated on a hypothetical example.*

Keywords. Agricultural production planning, stochastic programming model, decision support system, computer optimization

1 Introduction

Agricultural production is characterized to a great degree by uncertainty, which makes planning more difficult. Apart from the human and technical factors, agricultural policy measures and uncertain condition on the agricultural products market, this production is greatly influenced by different natural factors. If conducted in due time, there are activities that can alleviate negative impacts of some natural factors. Thus, for example, investments into irrigation and drainage systems can help us to avoid serious damage caused by drought or excessive rainfall, whereas crop rotation planning and adequate fertilization can improve soil quality. Given all that, there still remains a variety of factors in agricultural production that are not fully predictable, i.e. they are susceptible to change. It follows that the parameters representing these factors cannot be determined with certainty. In such conditions, applying deterministic models in agricultural output planning is not justified.

In order to improve decision-making in this domain, this paper proposes a stochastic programming model based on computer optimization. The decision support system improved in this way becomes suitable for usage in conditions of uncertainty. Because of its character, modern information technologies play an important role in this model.

2 Research methodology

The paper presents theoretically and on a simple hypothetical example the process of constructing and solving the decision–making model in agricultural production, which is founded on stochastic programming. Stochastic programming models assume that parameters are not known with certainty, rather, they are defined as random variables that follow one of theoretical distributions. In the proposed model it is assumed that random variables are normal distributed, and that at least one constraint is stochastically set. When solving the problem posed in this way, it is necessary to establish deterministic equivalents of all probabilistic constraints. In this way, we get to the non-linear constraints. Since solving this kind of a problem is highly complex, it calls for adequate computer support. In this paper the program package *Mathematica* was used. The solution of the posed problem ultimately determines the quantity of agricultural output which should be produced and offered to the market, under given limits.

3 Model formulation

Deterministic models assume that the parameters are known with certainty, which makes it possible to establish optimal solutions to the posed problems. In contrast, stochastic models treat their parameters as random variables, which makes it necessary to forecast the realization probability of the constraints in which they appear. A general formulation for stochastic programming models defined in this way, in case when the objective function is maximized, reads as follows:

$$maximize\ z = \sum_{j=1}^{n} c_j x_j$$

subject to

$$P\left(\sum_{j=1}^{n} a_{ij}^{*} x_j \le b_i^{*}\right) \ge 1 - \alpha_i,\ i = 1,...,m$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i,\ i = m+1,...,r$$

$$x_j \ge 0,\ j = 1,...,n$$

It is thus assumed in the model that parameters a_{ij}^{*} $(i = 1,\ldots,m,\ j = 1,\ldots,n)$ and right-hand side value of constraint b_i^{*} $(i = 1,\ldots,m)$ can be defined as random variables. The constraints that do not contain such parameters are treated as deterministic in this model.

With regard to parameters defined as random variables, when establishing deterministic equivalents of probabilistic constraints we need to differentiate between three cases.

In the first case it is assumed that parameters a_{ij}^{*} are defined as random variables that follow normal distribution, with mean μ_{ij} and variance σ_{ij}^{2}. A deterministic form of the ith constraint, stochastically posed in this way, reads as follows:

$$\sum_{j=1}^{n} \mu_{ij} x_j + K_i \sqrt{\sum_{j=1}^{n} \sigma_{ij}^{2} x_j^{2}} \le b_i$$

It should be noted that in the stated expression K_i represents a cumulative distribution function of standard normal distribution.

In the second case, it is assumed that the right-hand side value of constraint b_i^{*} is defined as random variable that follow normal distribution, with mean μ_i and variance σ_i^{2}. A deterministic equivalent of thus posed ith constraint is the following:

$$\sum_{j=1}^{n} a_{ij} x_j \le \mu_i + K_i \sigma_i$$

The third case appears when both parameters a_{ij}^{*} and the right-hand side value of constraint b_i^{*} are defined as random variables in ith constraint. Here it is also assumed that parameters a_{ij}^{*} are normal distributed, with mean μ_{ij} and variance σ_{ij}^{2}, and that the right-hand side value of constraint is normal distributed as well, with mean μ_i and variance σ_i^{2}. A deterministic form of the stochastic constraint posed in this way now reads as follows:

$$\sum_{j=1}^{n} \mu_{ij} x_j + K_i \sqrt{\sum_{j=1}^{n} \sigma_{ij}^{2} x_j^{2} + \sigma_i^{2}} \le \mu_i$$

If this proves necessary, when posing the stochastic programming model a decision-maker can combine different forms of probabilistic constraints.

The basic steps in constructing, solving and implementing the analyzed model are shown in Figure 1.

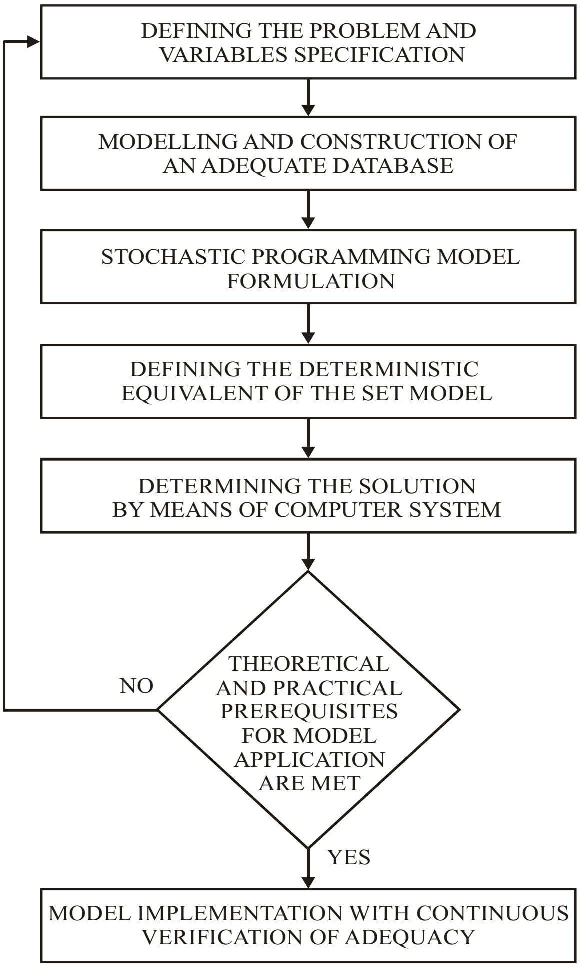

Figure 1. The basic steps in constructing, solving and implementing the analyzed model

In the first phase of model construction, it is necessary to define the problem and specify all relevant model variables. Experience of experts who have been involved in process of agricultural production can be of great help to managers in this step. Theirs experience enable managers to observe more precisely all the elements of analyzed problem.

Modelling and construction of an adequate database is the next phase in model building. This database should contain different data which is relevant for solving the problem. Some of them are data that refers to crops produced during the passed years, expected costs and incomes, and the available quantity of production materials. A modern information and communication technology allows manager to create, model, establish and use such database in relatively simply way.

After determining all relevant variables and theirs parameters, it is possible to formulate the stochastic programming model. As previously mentioned, in such a model some constraints can be defined as deterministic, but at least one has to be defined as stochastic. For all probabilistic constraints we need to define their deterministic equivalents.

Solving this kind of a problem without an adequate computer support is quite complex. It is the reason the program package *Mathematica* was used in this paper.

Before implementing the model, we need to check whether the theoretical and practical prerequisites for model application have been met. If the formulated model turns out to be inadequate, we have to review each phase in model construction, which also includes conducting the required modifications. In the forthcoming future the adequacy of the model should be continuously verified.

4 Numerical example

Let it be assumed that the management of an agricultural company wishes to determine the area, expressed in hectares (ha), which should be planted with peppers (x_1), lettuce (x_2) and tomatoes (x_3), and that the selling price has been predetermined with a wholesaler. Based on this price it is expected that the total income, including the incentives, will be 15000 EUR/ha for peppers, 19000 EUR/ha for lettuce, and 20000 EUR/ha for tomatoes.

Because of volatile market conditions, production costs cannot be determined with certainty, rather, they are defined in the model as random variables that follow normal distribution. It is thus estimated that the expected production costs for pepper will be 10500 EUR (μ_{11}) with variance 15000 EUR (σ_{11}^2), for lettuce 11000 EUR (μ_{12}) with variance 20000 EUR (σ_{12}^2), and for tomatoes 10000 EUR (μ_{13}) with variance 30000 EUR (σ_{13}^2). The total amount at disposal for producing the vegetables under consideration here is 260000 EUR.

Among other things, sulphate fertilizer NPK 5:20:30 is used to fertilize peppers, lettuce and tomatoes. Peppers require 600 kg/ha of that fertilizer, lettuce requires 5 kg/ha, and tomatoes require 500 kg/ha. In our example, it will be assumed that there are difficulties in the supply of this fertilizer due to production problems. For this reason, the parameter representing the quantity at disposal will be defined as random variable that follows normal distribution, with mean 9500 kg (μ_2) and variance 8100 kg (σ_2^2).

The agricultural company plans to cultivate no more than 25 ha with peppers, lettuce and tomatoes. To meet the contractual obligations to the wholesaler, peppers must be planted on 5 ha at least, whereas the area under tomatoes can be maximally twice as large as the one under lettuce.

It will be assumed that the first stochastic constraint will be realized with a minimum probability of 90%, and the second with a minimum probability of 95%. The stochastic model of the problem thus defined, with the objective to maximize total income, reads as follows:

$$max\,imize\ z = 15000x_1 + 19000x_2 + 20000x_3$$

subject to

$$P\left(a_{11}^{*}x_1 + a_{12}^{*}x_2 + a_{13}^{*}x_3 \le 260000\right) \ge 0.9$$
$$P\left(600x_1 + 5x_2 + 500x_3 \le b_2^{*}\right) \ge 0.95$$
$$x_1 + x_2 + x_3 \le 25$$
$$x_1 \ge 5$$
$$-2x_2 + x_3 \le 0$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

The deterministic equivalent of the set model is the following:

$$max\,imize\ z = 15000x_1 + 19000x_2 + 20000x_3$$

subject to

$$10500x_1 + 11000x_2 + 10000x_3 + 1.282S \le 260000$$
$$S = \sqrt{15000x_1^2 + 20000x_2^2 + 30000x_3^3}$$
$$600x_1 + 5x_2 + 500x_3 \le 9500 + 1.645\sqrt{8100}$$
$$x_1 + x_2 + x_3 \le 25$$
$$x_1 \ge 5$$
$$-2x_2 + x_3 \le 0$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

Determining a solution to the resulting model is inconceivable without computer systems. Their usage is one of the basic features of decision support systems, because they incorporate information technology in order to advance the process of solving the problem. In this way computer systems improve the decision-making process. For this purpose in this particular case we used program package *Mathematica*. This application allows us to easily solve a different kind of nonlinear optimization problem. Figure 2 shows *Mathematica's* onscreen display of analyzed problem.

The program package *Mathematica* has yielded the following solutions: x_1=5, x_2=6.5885 and x_3=13.177. With these areas under cultivation, it is expected that the agricultural company will achieve the income of 463722 EUR.

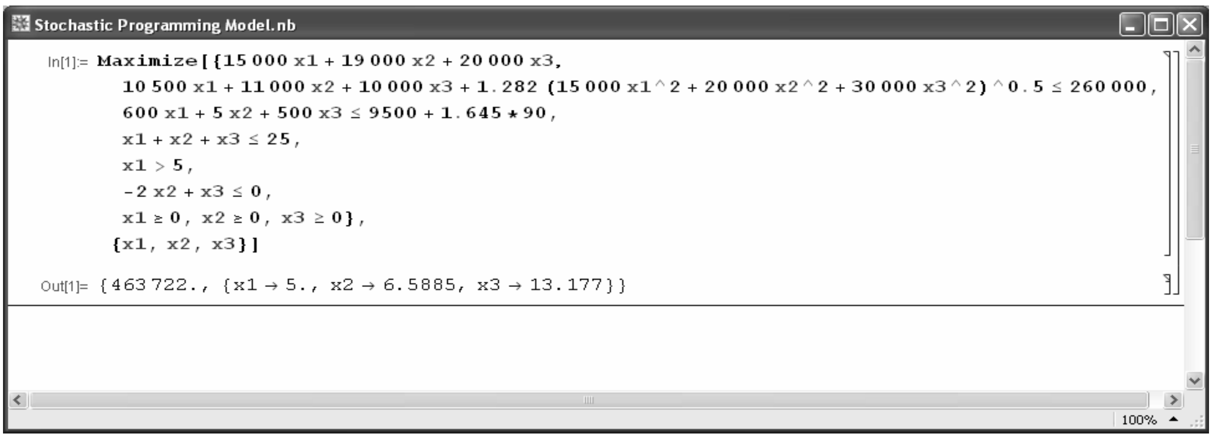

Figure 2. *Mathematica's* onscreen display of analyzed problem

5 Conclusions

Appropriately implemented stochastic programming models, which are intended for use in conditions of uncertainty, can be a powerful tool for agricultural management in production planning. Including such models into a decision support system that deals with the type and quantity of agricultural output to be produced and offered to the market can improve the whole management process. In addition to the adequate quantitative model, decision support systems incorporate also the information technology required for solving, analysis and modification of the set model. Without computer support it would be virtually impossible to solve a problem of non-linear optimization, obtained after establishing deterministic forms of stochastically set constraints. Modern program applications are so easy to use that even the managers who are no computer experts can apply the proposed stochastic programming model without much difficulty.

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