Computer-Aided Simulation Model of Smoked Meat Inventory with Price Breaks

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Abstract. *Computer systems provide indispensable support in constructing, solving and implementing simulation models. This paper discusses a smoked meat inventory model with simulated demand. In order to make a more precise estimate of the demand, the proposed model implies the use of the beta distribution. Simulated demand values, generated by means of a computer, represent the inputs in the inventory model with price breaks. This model is characterized by taking into account the possibility of decreasing the purchase price when the ordered quantity is increased. In addition, the management needs to consider the fact that unsold smoked meat products past use-by date directly increase the costs and thus reduce store operation efficiency. In this model, for each simulated demand rate the order quantity is determined which results in minimum overall costs. The optimum order is thought to be the one which has been determined most frequently within the conducted set of simulations. In order to increase the model's use value, the decision-making process based on its use has been shown in a flowchart. Furthermore, the paper proposes an adequate computer program for solving the model, written in LINGO's modelling language.*

Keywords. Smoked meat inventory model, order quantity, computer-aided model, simulation, LINGO's modelling language, computer program

1 Introduction

When managing the inventory of smoked meats in stores that carry such products, the main concern is to ensure permanent supply of sufficient quantities that will satisfy the demand, while minimizing total inventory costs. In all this, a special problem is the limited shelf life of these groceries, i.e. the costs related to their possible decay. This is why the management needs to make careful estimates of demand when planning the inventory. Their decision-making can be significantly improved by using simulation models. With this goal in mind, the paper presents a model in which the demand for a certain type of smoked meat is defined as a random variable which follows the beta distribution. Its parameters can be estimated on the basis of previous sales results. It follows that creating an adequate database is a crucial step in constructing the model.

Demand values, generated from the beta distribution by means of a computer and adequate software, are used in the inventory model with price breaks to calculate the optimum order quantity. A specific feature of this inventory model is that it allows the possibility to decrease the purchase price of a product with increasing order quantities, which is common in trading practice. By granting quantity discounts, manufacturers stimulate the traders to buy bigger quantities of their products. In such a situation, managers have to assess whether the benefits of ordering larger quantities outweigh the costs of storing these products and risks related to their possible decay. When solving the model, demand values are one element to be determined, the others are ordering cost, unit holding cost, and purchasing cost, which vary depending on the quantity ordered.

2 Methods

The simulation model presented in this paper is based on an inventory model with price breaks. Given that the demand for smoked meats has been defined as a random variable in the model, it belongs to the group of stochastic models. Construction, solving and implementation of the proposed model are based on the use of computer systems. To make the model understandable to a greater number of users, the decision-making process based on the model is shown in an adequate flowchart. By quoting the computer program, which was written in LINGO's modelling language, we tried to emphasize the use of information technologies in solving the stated problem, i.e. the need to create an adequate decision support system.

3 Overview of previous research

Inventory management has an important role in many companies. This management function has been successfully treated by quantitative methods. Numerous inventory models have been developed in operations research during the passed decades. Their mathematical formulations, including this one for inventory model with price breaks, are available in many publications (W.J. Stevenson [11], C.P. Bonini, W.H. Hausman, H.Bierman, Jr. [3], F.S. Hillier, G.J. Lieberman [5]). The quality of decision-making in inventory management by using of this models could be significantly improved by simulations (N.R. Osowski [8], C.D. Lewis [6], G. Dukić, D. Dukić, M. Sesar [4]). Our paper continuous previous research in this field, by focusing on the smoked meat inventory. A specific feature of the proposed model is the demand values simulation based on a random variable generated from the beta distribution.

4 Model formulation

An inventory model allows the management to determine the order quantity with minimum total costs. In an inventory model with price breaks, they consist of three components: ordering cost, holding cost and purchasing cost. Ordering costs are all the expenses which appear as roughly constant values in the process of ordering smoked meats, such as transportation costs, handling and insurance costs. Holding costs, which include among other things energy costs, stock-taking, and the costs associated with meat decay, encompass all the expenses connected with the storage of such products. Given that the increase in order quantity reduces the unit price, purchasing cost is definitely an integral element of the model being analyzed here. It follows from the above that total costs (C) can be calculated in the following way:

$$C = \frac{K}{q}x + \frac{H}{2}q + P \cdot x \qquad (1)$$

Where
K = Ordering cost for one order
x = Demand rate per unit time
q = Order quantity
H = Unit holding cost per unit time
P = Unit price

The optimum order quantity is obtained by deriving the total cost function with respect to variable q and equalized the obtained partial derivation to zero:

$$\frac{\partial C}{\partial q} = -\frac{K}{q^2}x + \frac{H}{2} = 0 \qquad (2)$$

Thus, if the price breaks effect were to be disregarded, the formula for determining the optimum order quantity, deduced from the above equation, would read as follows:

$$q = \sqrt{\frac{2Kx}{H}} \qquad (3)$$

Product unit costs, which depend on ordered quantities, have an impact on moving the total cost curve for different amounts. The lowest total cost curve reflects the lowest unit price. This relationship, which assumes three different levels of unit prices (P_1 = highest, P_2 = medium, and P_3 = lowest), is shown in Figure 1. It should be noted that each total cost curve is valid only for the quantity range that a particular unit price refers to.

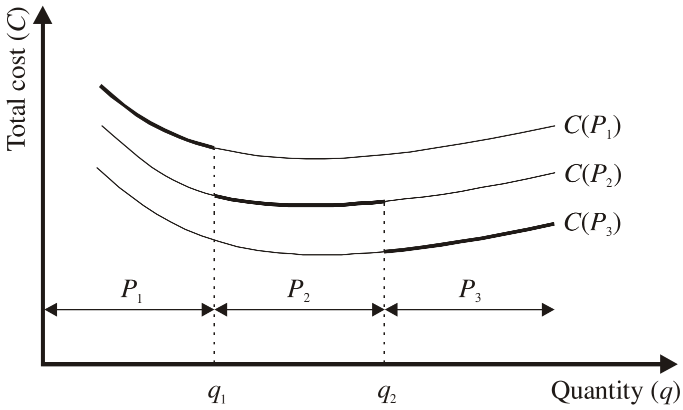

Figure 1. Variation of total cost curves depending on price breaks

In case when holding costs are constant, i.e. independent of a product unit price, all the curves representing total costs will reach their minimum points at same ordered quantity (Figure 2).

The solution of the model consists therefore in finding the total cost curve within whose quantity range the determined economic order quantity falls. If that quantity is valid for the lowest total cost curve, then the global optimum is achieved, and further calculation is not required. If, however, the determined quantity does not fit into the quantity range presented by the curve referring to the lowest unit price, it should be examined whether this has been achieved for any of the other curves. This means that total costs arising from all higher volume orders need to be calculated. Ultimately, the optimum order quantity is deemed to be the one for which minimum total costs have been determined. In the examination process, it is sufficient to take into account only those quantities where a price break occurs, since it is only in these points that an optimum solution can be determined.

Given the unsteady demand for smoked meat products, the model described here can be significantly improved by defining demand as a random variable which follows a certain theoretical distribution. In this paper the use of the beta distribution was assumed for this purpose. Defining the demand for smoked meat products as a random variable which follows the beta distribution can be justified by the characteristics of this distribution.

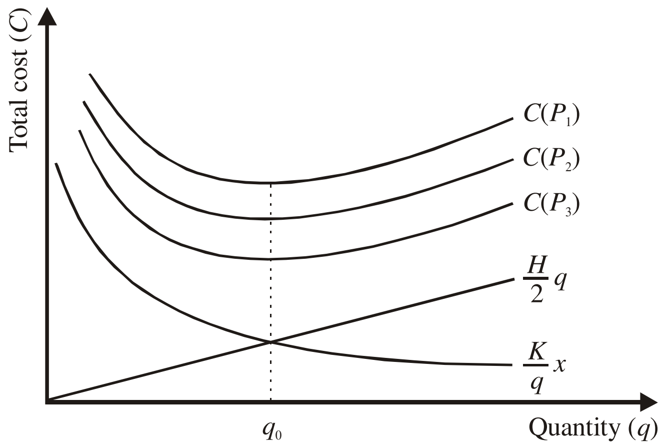

Figure 2. Graphic solution of inventory model in case of constant holding costs

The probability density function of the beta distribution, with parameters α and β, is:

$$f(x) = \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}},$$

$$a \le x \le b\,,\ \alpha,\beta > 0 \qquad (4)$$

In the above expression $B(\alpha,\beta)$ represents the beta function:

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt \qquad (5)$$

Before the simulation process itself, we need to estimate the lowest (a) and highest (b) values of demand for the analyzed smoked meat during a particular time interval. It is advisable to found this estimate on the available data from a previous period. In the process of constructing the proposed model it is therefore important to create an adequate database. For the demand distribution formed in this way, the value of mean ($\bar{x}$) and variance (s^2) are calculated, which are then the basis for estimating the parameters α and β:

$$\alpha = \left(\frac{\bar{x}-a}{b-a}\right)\left(\frac{(\bar{x}-a)(b-\bar{x})}{s^2}-1\right), \qquad (6)$$

$$\beta = \left(\frac{b-\bar{x}}{\bar{x}-a}\right)\alpha \qquad (7)$$

Once the parameters α and β have been determined, the first random number is generated, which represents the probability of demand for the analyzed product. The pertaining value of beta distribution is then determined for this level of probability. The demand value generated in this way is used in the model to calculate the optimum order quantity.

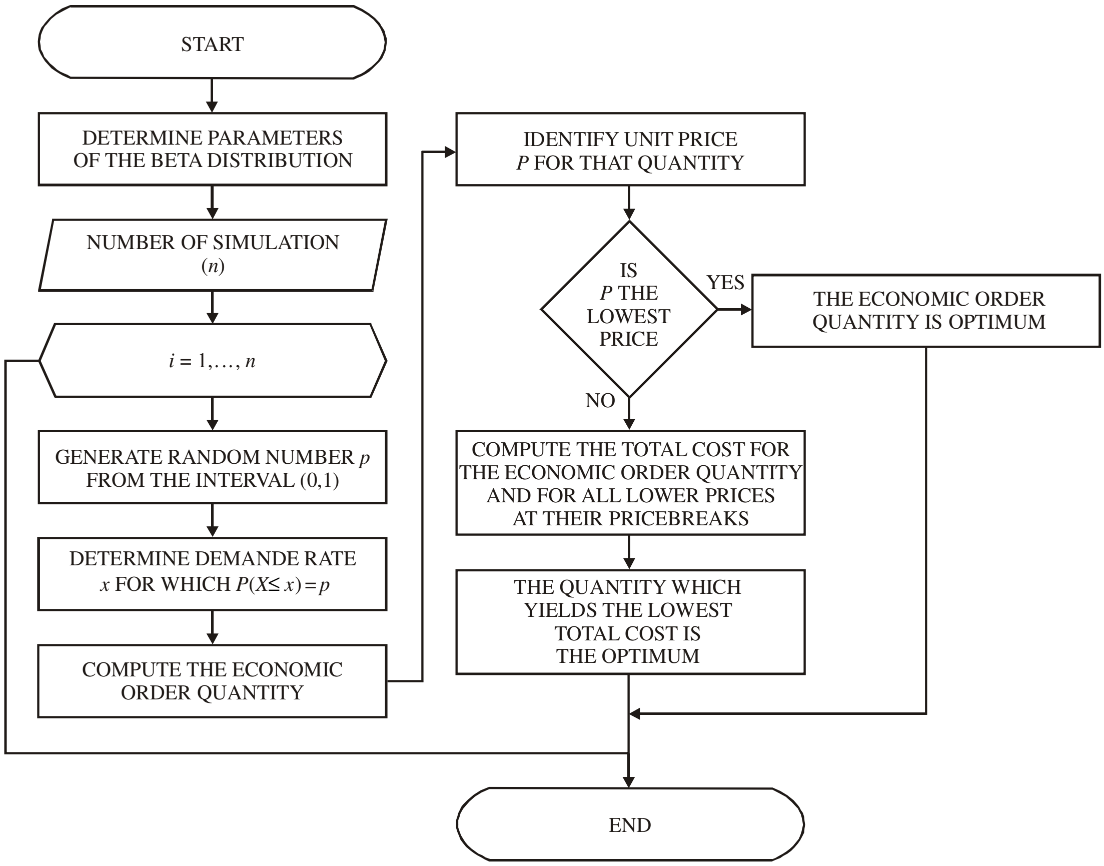

Figure 3. The process of determining the solution using the proposed model

Since the order quantity depends on demand, it is to be expected that process repetition will result in determining different solutions. Ultimately, the optimum order quantity will be the one which has been determined most frequently within the conducted set of simulations. In order to reduce the possibility of error, the described procedure should be repeated until the optimum value can be unequivocally singled out.

Figure 3 shows the flowchart which summarizes the decision-making process based on the proposed model.

Implementation of computer systems in the area of management has significantly simplified and accelerated the procedure described here.

5 A Hypothetical example of constructing and solving the model

Let it be assumed that the management of a supermarket aims to determine the optimum order quantity of a certain type of smoked meat during one month period. Let it further be assumed that its ordering cost is $K = 150$ EUR, and its holding cost is $H = 2$ EUR/kg. If the supermarket orders up to 300 kg of this product, the price will be 15 EUR/kg, for a purchase between 300 and 700 kg the purchase price is decreased to 14 EUR/kg, whereas for orders exceeding 700 kg the price is 13 EUR/kg. The producer can deliver no more than 1000 kg of that particular product. Therefore, the price break points are $BP_1 = 300$, $BP_2 = 700$ and $BP_3 = 1000$, while the accompanying unit prices are $P_1 = 15$, $P_2 = 14$ and $P_3 = 13$.

On the basis of available sales data it has been established that the minimum monthly demand is $a = 200$ kg, with the maximum being $b = 450$ kg. It will be assumed that the sales distribution mean is $\bar{x} = 300$ kg, and the variance $s^2 = 2500$ kg, in which case, according to (6) and (7), $\alpha = 2$, and $\beta = 3$.

In the simulation process, the first random number 0.4631 was computer generated. To this probability of the random variable X, which follows the beta distribution with the stated parameters, corresponds the demand value $x = 291.1821$ kg, i.e.:

$$P(X \leq 291.1821) = 0.4631$$

To solve the above stated hypothetical problem we have used the *LINGO* application. The corresponding computer program in LINGO's modelling language is:

```
MODEL:
SETS:
  RANGE/1..3/:
  BP, P, EOQ, q, TC;
ENDSETS
DATA:
  x = 291.1821; K = 150; H = 2;
  BP = 300, 700, 1000;
  P = 15, 14, 13;
ENDDATA
  @FOR(RANGE: EOQ = (2*K*x/H)^.5);
  q(1) = EOQ(1) - (EOQ(1) - BP(1) + 1) *
         (EOQ(1) #GE# BP(1));
  @FOR(RANGE(J)| J #GT# 1:
  q(J) = EOQ(J) + (BP(J-1) - EOQ(J)) *
         (EOQ(J) #LT# BP(J-1)) -
         (EOQ(J) - BP(J) + 1) *
         (EOQ(J) #GE# BP(J)));
  @FOR(RANGE: TC = K/q*x + H/2*q + P*x);
  TCMIN = @MIN(RANGE: TC);
  QOPT = @SUM(RANGE: q*(TC #EQ# TCMIN));
END
```

With all the above assumptions, the optimum order quantity is 300 kg of the product. In this case, total costs amount to 4522.14 EUR. After the procedure of demand simulation and model solving has been repeated 150 times, the optimum order of 300 kg of the product was determined 102 times, i.e. in 68% of cases, which indicates that this is the best solution.

6 Conclusions

The paper has presented a computer-aided simulation model of smoked meat inventory with price breaks. In its construction, special attention was given to the issue of correct estimate of demand for this type of product. If the demand is properly determined, traders can avoid losses related to the unsold quantities with past use-by date, but also opportunity costs arising from insufficient inventories of such products. The specific feature of the proposed model is demand rate simulation from the beta distribution. Information technologies play a crucial part in its construction, solution and implementation. Without those technologies it would be virtually impossible to efficiently carry out all the required procedures.

This paper continuous previous research in the field of the smoked meat inventory. The model presented in the paper is an abstract construction of reality, which means that the uncertainty in the area of inventory management cannot be completely avoided. If some relevant variables are left out from the model, if their values are misjudged, if there is no flexibility in relation to the changes in the environment – these are just some of the reasons that can decrease the use value of the model. On the other hand, if such deviations are noticed in time, and then corrected, the model can become a powerful tool in inventory management.

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