Empirical Learning Styles in Simulation of Investment Game

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Abstract. *Environment in which an individual makes decisions may be quickly changing, i.e. volatile, causing a change in perception as far as available versions of decision are concerned. By reaching decisions through time, an individual learns based on personal experience, i.e. based on feedback on possible and actual results of relevant decisions. For the purpose of this research, we developed a game containing elements of a strategy and allowing a simulation of decision-making by a number of individuals. Empirical testing on a sample of players was to determine styles of learning and decision-making. As far as choices of strategy are concerned, it was determined that there is a significant sensitivity of players toward negative changes of payoff. Reactions to "missed opportunity" modeled according to Camerer's Consideration Index are shown through various intensities of changes in inhabiting of successful and unsuccessful strategies.*

Keywords. Consideration index, decision-making, empirical learning, payoff, reduced backward information, strategic choice

1 Introduction

For the past several years, there has been a great emphasis in the domain of strategic management and organizational theory on organizational design solutions in form of learning organizations. Research efforts and concepts related to this area are already becoming established and include studying cognitive models, system approach, teamwork, etc. [12], [1]. The phenomena of knowledge management, as well as supporting drivers of learning organizations are studied significantly, but less attention is given to learning styles [10]. Therefore, our motivation and expected contribution is oriented to performing research of styles of empirical learning according to game theory models. The rationality of decision makers is expressed by their aspiration to maximize the profit, according to available decision alternatives and criteria. There are some scenarios in which, due to lack of information, all decision variants are virtually "equally interesting". It is difficult in these situations to make decisions according to principles of rationality, because participants are almost without any orientation. This is the exact principle our business game was modeled on and used in our research related to empirical learning. Decision-making and learning situations of our research do not fully contain properties of strategic games, but their context is very similar. That analogy enables the usage of behavioral game theory learning models for the purpose of analyses. We claim that an integral approach connecting different decision-making and strategic learning disciplines is very important for various scenarios and games experimentally performed for this paper. We have, therefore, illustrated and specified elements of this multidimensional framework in the chapters below.

2 Uncertain scenarios and game theory

Decision-making is defined as choosing a direction or a course of implementation among several alternatives. It is the process of creating and evaluating alternatives and the process of choosing among multiple solutions [13, 9].

In order to understand and deal with scenarios we have created in the investment simulation game, it is useful to present relevant issues of decision-making theory. Literal and subjective rationality that reaches solutions based on of limited knowledge possessed by individuals finds their causes in restrictions of time and costs, as well as imperfections of available information. Constraints in decision-making often reduce decision-making choices to a smaller number of alternatives, but these limitations can be obstacles for rational decision-making, especially if they take on forms of decision-makers' personal constraints (i.e.

preconceptions), inclusion of favorite alternatives, creativity deficiency, and so on [14].

When numerous alternative decisions can be chosen in practice, it is advisable to reduce the risk included. The algorithm used to achieve this task is in certain decision disciplines called 'mixed strategy usage'. Managers' attitude toward risk is interpreted in numerous publications and proverb that "eggs should be kept in more than one basket" can be found with a number of prominent authors [8]. Decision-making in conditions of uncertainty represents an unwanted scenario, characterized with: lack of information, insecurity of available information, and inability of situation assessment decision-making. In uncertainty conditions available alternatives do not have clear attributes as the basis for the players to perform evaluations according their preferences. Therefore, in such scenarios, those created as well as through our simulation, the well-known algorithm of multi-attribute evaluation for support in decision-making is not possible [7].

It is important to stress that effective decision-making process should satisfy several criteria, among which are two claims related to information: provision of information and relevance of information [6]. Decision-making in practice describes a process that containing acquisition and manipulation of information stemming from the environment. In our created model, the level of information directing decision-making is extremely low and, according to the theory of information, the uncertainty in knowledge of the input (attributes of strategies) as well as the output (results of strategies and their attributes) is very high.

2.1 Game theory

Special attention regarding uncertain scenarios belongs to an interactive decision theory- the game theory. The mission of game theory is to search for solutions (optimal behavior) in circumstances of competitiveness, of interaction among two or more players with different interests. The game theory is actually a rational decision theory for interactive situations. Equilibrium points in games can usually be defined through the Nash equilibrium concept and represent solutions of games that determine optimal strategies for each game player. For a great number of known games, the solution exists in the form of mixed strategy. Therefore, participants should choose strategic options with certain frequency [11]. In the context of our investment game, use of mixed strategies carries a different meaning and primarily pertains to reducing the risk level.

Extension of the game theory as a mathematical discipline, by introducing psychological moments, leads toward a situation in which game theory is no longer a model for rational conflict. This theory becomes behavioral theory, but real behavior could not be explained exclusively on basis of rationality. In the decision-making process, the player must take care of other participants, as well as characteristics such as: belief, cooperation, suspicion, reciprocity, and repetition.

The game theory can be conceived as a mathematical treatment of scenarios in games among different players with specific cognitive capabilities [3]. The empirical investment game we created possesses preconditions for processing with behavioral learning models. Investigations of rules in players' decision-making in empirical situations have a very rich history. One such effort is noted in [4]. Behavioral game theory approach for consideration of strategic thinking and learning examples offers valuable knowledge for the domain of organization and strategy. Also, players learn about types (styles) of decision making and acting, not which specific strategy in a certain game is more attractive/better.

3 Learning based on experience

To realize what real people do in strategic situations, how they make and change decisions is desirable in a number of diverse research areas. Mathematical formalism of the game theory in combination with behavioral and evolutionary concepts creates one powerful tool in supporting our dealing with strategic scenarios.

3.1 Behavioral game theory - strategic learning

Behavioral game theory includes concepts and experimental proofs for the purpose of development of understanding strategic operations, and it is useful for analyzing interactions in business, politics and society [3]. It is about understanding players' mental models that changed during interactions in the game. Some of the most prominent segments of behavioral game theory are learning theories. In empirical situations learning represents a change in behavior based on history of successful and unsuccessful attempts. Players have limited rationality and could generally not make optimal decisions right away. According to this approach, three theories of learning exist [3]:

a) Learning through repeating actions with positive outcomes; in the case of negative results, actions are not repeated (*reinforcement*);
b) Learning on the basis of developing assumptions about what the others will do by tracking history of their behavior. Strategies are chosen according to these assumptions (*belief learning*);
c) Learning based on past experience - Experience-Weighted Attraction (EWA). The general idea here is that strategies possess different levels of attraction for

decision makers. This theory includes concepts of models a) and b).

EWA explained and predicted what people actually choose in games more accurately than either of the other theories. Created by Teck Ho and Camerer C.F. (1999) the experience-weighted attraction model has two variables: attractions and experience weights. Strategies in games have their numerical evaluations, so called «attractions». Players updated these attractions after every period of experience, and start at $A_i^{\,j}(0)$ [3, 305]. Labels i, j are devoted to *player i* and certain *strategy j*. All other participating players are marked with *(-i)*. Experience weight $N(t)$ starts at initial value $N(0)$ and weakly increasing according to:

$$N(t) = \phi(1-\kappa)\cdot N(t-1)+1 \qquad (1)$$

ϕ and κ are parameters. The payoff for *player i* when he plays *strategy j* and other players choose their particular strategies is given with $\pi_i(s_i^{\,j}, s_{-i}(t))$. Attractions are updated according to [3, 305]:

$$A_i^{\,j}(t) = \frac{\phi * N(t-1) * A_i^{\,j}(t-1)}{N(t)} + \frac{[\delta + (1-\delta)\cdot \mathrm{I}(s_i^{\,j}, s_i(t))]\pi_i(s_i^{\,j}, s_{-i}(t))}{N(t)} \qquad (2)$$

In modeling the parameters of strategic learning, for our investment game, we accepted definitions of these models as well as learning indexes that are explained later. Parameter δ is weight factor related to the opportunity cost. Parameter ϕ illustrates the level of former actions neglecting. Duration of experience relevance is important regarding the speed of changes in environment. Parameter κ takes value in interval [0, 1]. It is zero when players update strategies in accordance with their earnings, and it has value 1, when is about cumulative choice.

Parameters are estimated for different types of games and as well in a «p-Beauty contest game» for EWA model there is $[\ \phi, \delta, \kappa\ ] = [0.0; 0.9; 0.0]$. For so-called «continental divided game» estimation is that $[\ \phi, \delta, \kappa\ ] = [0.61; 0.75; 1.00]$, according [3, 313].

From one another perspective, the EWA learning theory determines the mathematical method for past experiences matching by means of indexes of **consideration, change** and **commitment** (CCC indices).

The EWA theory specifies a precise mathematical way in which past experiences are combined using the consideration, change, and commitment indices. These indices correspond to different learning styles [2]. These indices constitute the main focus of our research.

The *Consideration index* represents one measure of relative importance (weight) that people gave to miss opportunities or results in their previous decisions. Decision makers have capabilities of understanding the values of the unrealized opportunities. That can be a firm learning factor in certain situations [2]. Economists often refer to the value of a lost opportunity as "opportunity cost". When the consideration index was higher, managers focused more quickly on the strategies they wish they had chosen, minimizing their missed opportunity regret. A decision maker who always changes his opinion and indicates mistakes has a high consideration index.

3.2 Modeled investment game

During decision-making, information is processed, i.e. collected, classified and grouped according to specific criteria, reduced and evaluated. An important phase is also that of information interpretation. Within the behavioral experimental model area, it is important to emphasize research related to the importance of feedback information from the known scenarios of the game theory [9], [15]. Therefore, the game modeled in this paper contains characteristics of somewhat reduced feedback through the decision-making process and forms a sort of continuation within the domain of empirical learning simulation.

In our research, sample of thirty people from students population are engaged to participate playing this game. In this game fifteen players simultaneously occupy networked computers using the instructions given to them by the game coordinator, who indicates each following step and limits the decision-making time. Before starting the game, the participants know the basic game rules used in order to achieve the best possible score in the 27 steps of the game. Players simultaneously choose one of 20 strategies in each step carrying a 1-minute time limit.

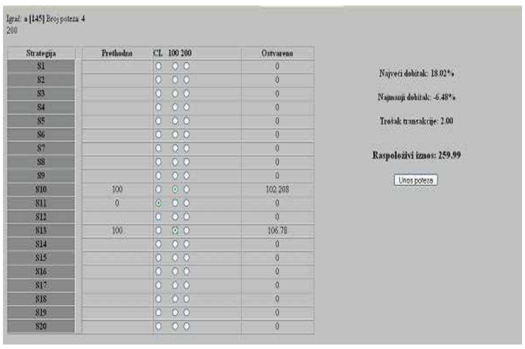

Figure 1. Investment game screen – example of situation

The strategies represent the chosen stock from the Croatian stock exchange listing with their time-related dynamic shown through the period of 27 weeks in 2007. Percentage of the stock value change represents the profit (loss) realized through the use of the individual strategy and the amount is added to the player's account. Each player always invests the same amount (200) in each step. This amount can be allocated to two strategies at the most. The change of strategy is penalized with 1%.

The modeled game represents an extreme version of the typical strategic game in which the individual has no information about his adversary. He does not know the history of their strategic choices, but he must choose the strategy (without the attributes) in order to optimize his effect. The only return information (feedback) given through the game is related to the maximum/minimum payoff of the last two rounds. Therefore, our game does not include the so-called "updating belief" phenomenon, and we can conclude that the referred learning model is not adequate for us. However, two important strategic concepts are very adequate. In given situations, the player experiences the context of considering the "missed opportunities", i.e. he experiences the perception of his payoff in relation to the best possible payoff. This fact can direct the decision maker to change or keep his strategy.

In addition to focusing on the mechanisms related to missed opportunities, our intentions are focused on identifying the situations of "reinforcement" approach: another known approach in empirical learning. The main purpose is to analyze how positive results, connected with specific strategies, support the player's fidelity to these strategies and vice-versa.

4 Game results – learning styles

In our research, related to the learning problem, we have accepted the considerations of the EWA developers and authors about difficulties in practical usage of original relations based on their high complexity.

Especially, concerning the our research interest in CCC indices, and with respect to game structure and characteristics, we prefer strategic learning treatment by mean of assessments of consideration index and certain others relevant learning and deciding parameters. Our aim was not in domain of adjusting completely to formalism given by eq. 2.

The first variable we come across is the difference between the average profit of used strategies and the average achieved results of game participants in each step. This dynamic was the focus of our research and analysis. We found that, through time, there is a recognized albeit small trend in reduction of this difference. The reduced feedback is not adequate for the players and their learning, i.e. their pursuit of improving their strategic decisions and, in time, surpass the level of the average payoff.

In our model, attractions for strategies are updated according to: previous payments, trends of strategies goodness changes, and perception of missed chances. Players have certain information about their past or current strategy, but they don't know about the other strategies Because of that, they estimate only one strategy and make decision about its attraction-whether to keep this strategy or to change it.

Another reason why there was no domination of payout to the players in relation to the average payouts must be related to variable success of strategies.

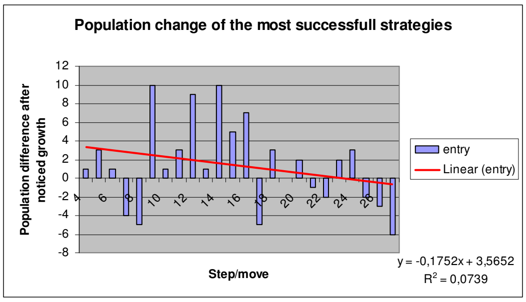

Figure 2. Reaction speed – population of increasing strategies

Our next goal is to define the reactions of the players due to the changes of chosen strategy success (environment). This indicator is especially interesting for the variable environments requiring fast adjustment. To that end, we examined the reaction speed of the players, i.e. we defined 5 strategies with the highest growth and 5 strategies with the highest reduction and did this for three steps in a row. Then we counted the players for each of these strategies before and after the high increase/decrease they realized.

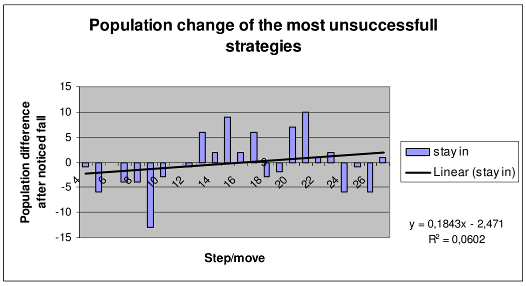

Figure 3. Reaction speed - abandoning decreasing strategies

The results are shown on Fig. 2 and Fig. 3. We can see that the trend is positive across the board for successful strategies, meaning the payoff increases. Toward the end of the game circumstances change through the assumption of the cause being the change in strategy payoff. This particularly pertains to step 18. and further, when player behavior changes, and average earning by strategy becomes lower. Consistency in behavior could be seen on the graph that gives the difference in succession of the most unsuccessful strategies. Cumulatively, the result of the most successful strategies is +33 (entries), while the result of the most unsuccessful strategies is -4 (exits).

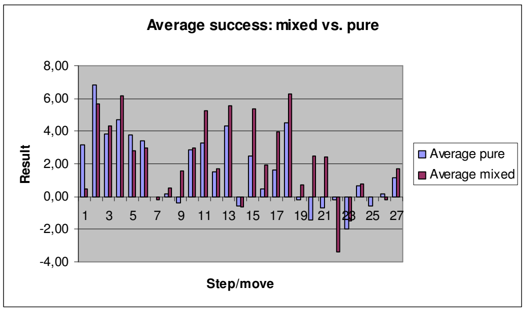

Figure 4. Success of mixed and pure strategies.

The next question we tried to answer is about frequency of choosing mixed strategies being connected to success. For that purpose, we determined the frequency of choosing mixed/pure strategy in each step by each of the players. Then we determined the average success of players who played "mixed or pure". The results are shown in table 1 and Fig. 4. The average success for mixed strategies is 2.30, and for pure strategies 1.59. Frequency of choosing mixed strategies and relaxing risk is relatively high - 60% of strategic choices are mixed strategies.

Table 1. Frequency of choosing mixed and pure strategies.

	Pure strategies	**Mixed strategies**
Frequency of choosing strategies	11.07	16.93
Standard deviation	2.51	2.51
% frequency	40%	60%

While modeling so called **consideration index**, in order to determine the styles of strategic learning, and also for player's reaction on their perception of missed opportunities, we used an approximate approach. We also changed the demanding EWA attraction calculation with calculation that considered 25% of the most successful and 25% of the least successful players. For each step, we detected the 7 most successful and 7 least successful players.

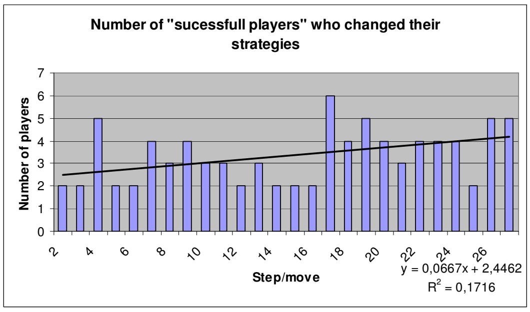

Figure 5. Perception of missed opportunity at low payoffs.

In the next step we tracked these same players and noted the number of those who changed strategies (in both groups). Since players receive feedback about the amount of highest payoff, that fact should motivate unsuccessful players to change their strategies.

Sensitivity related to the consideration index with these players should be more accented. Related to expected changes of strategic choices, the results are shown on Fig. 5 , Fig. 6, and table 2.

Table 2. Reaction comparison reaction - perception of missed opportunity

	From higher achievement group	From the lower achievement group
Sum of changes	87	161
Average (number of players)	3.35	6.19

It seems that players understand strategies consistently. Those who achieved payoffs close to the highest payoffs for their strategies were far less motivated for change. Players with lower payoffs changed their strategies more frequently.

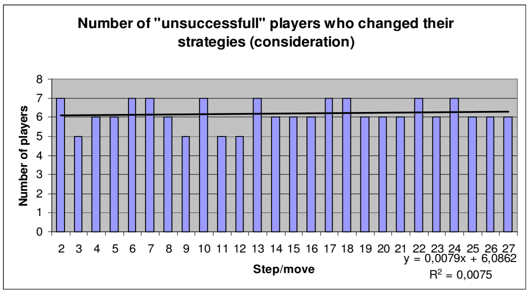

Figure 6. Perception of missed opportunity at low earnings.

Players consider "missed opportunities" and proportions in relation to success are indicative. Their perception of strategies is strictly patterned; they do not perceive them as options with totally stochastic payoffs.

Conclusion

Studying experience learning and specific player styles is an activity that can provide some usable answers to questions relevant for modern organization and strategic management. The investment game was developed with the special feature of reduced feedback for participants. Although not containing all the attributes of a strategic game, it is subjected to analyses based on formalism of the behavioral game theory and their learning models. The game theory system, adapted to our conditions, offered interesting answers concerning behavior of "reduced rationality" decision-makers trying to learn through experience.

Players demonstrated a certain attitude of loyalty to strategies and their consistent but rigid perception. According to Camerer's Consideration Index, players with less successful strategies were exceptionally open to changing their choices. The consideration index of learning that refers to missed opportunities obviously has an important role in the simulated game, and the players were quite careful in their perception of "distance" from the best payoffs. Investigating speed of reaction to changing effectiveness of strategies has shown that the players were recognizing the most successful strategies through time, since they carried a positive yield. The participants showed a tendency of playing mixed strategies, and that choice was shown to be more successful on average than the choice of pure strategies.

Future work

Further investigations in the domain of learning and decision making by experience is planned to take several possible directions. It should be emphasized that a formalism was developed in the domain of so-called evolution games describing expected behavior of particular population (players), especially regarding defined balanced game states, i.e. the goal players wish to achieve through time. Concepts of the evolution game theory are useful as a framework for analyses of player's decision making/behavior in a simple game that we developed on the bases of investing models. Regularities in players' decisions using the evolutionary approach refer to dynamic system properties [5]. The next course of possible effort is constructing a model with an option of increasing feedback used for decision-making. For this circumstance, possibilities of comparison between success and other factors hold significance in two scenarios. Another variant refers to attaching attributes to strategies, whereby the players could, by analogy to processes of multy-criterion evaluation, make their choices according to their own preferences.

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