Optimisation of portfolio of stocks at ZSE through the analysis of historical data

Robert Fabac, Dušan Munđar
Faculty of Organization and Informatics
University of Zagreb
Pavlinska 2, 42000 Varaždin, Croatia
{robert.fabac, dusan.mundjar}@foi.hr

Abstract. *Decision-making of investors at the stock exchange can be based on the fundamental indicators of stocks, on the technical indicators, or can exist as a combination of these two methods. The paper gives emphasis to the domain of technical analysis. In the broader sense the technical analysis enables the dynamics of the expected future values of the shares estimation. This can be performed on the basis of the data on historical trends of the revenues, profits and other indicators from the balance sheet, but also on the basis of historical data on changes in the values of the shares. Companies generally belong to the different sectors that have different presumptions of development resulting from the global market trends, technology and other characteristic. Processing of historical data values of the outstanding shares of ZSE is origination of this research. Investors are interested to know the estimation of future returns for the stocks as well as the size of the risk associated with the expected returns. Research task in this paper is finding the optimal portfolio at the ZSE based on the concept of dominant portfolio by Markowitz approach. The portfolio is created by solving non-linear programming problem using the common software tools. The results of obtained optimal portfolios contain relevant conclusions about the specifics of the shares as well as the characteristics of the industrial sectors but also provide a further knowledge about diverse sectors treatment at the stock exchange in a multi-year period.*

Keywords. stocks, historical data, Markowitz Portfolio Selection, economic sectors, Zagreb Stock Exchange, expected yield, risk

1 Introduction

Complete approach toward the companies' stocks evaluation includes indicators of current business and financial successfulness as well as the perception of future movements. One integral model of investor's decision making should be structured to contain several dimension- evaluating of fundamental stock's ratios, analyzing technical indicators, comprehension broader business environment and assuming future trends in national and international economy.

By technical analysis in a wider meaning, expected dynamics of future stocks values can be created on the basis on data about historical movements of earnings, profits and other balance sheet indicators, but also on the basis of data about historical changes of the stock values. For the reason of gravest historical data analysis approach, it is required to take as long as possible time period in computation. The circumstances regarding the ZSE are not very convenient since it has short active history. In past several years at least two very unusual phenomena occurred at our stock exchange: spectacular market growth (2003.-2007.) due to accessing to international alliances and severe drop of all market indices caused by great world economic crises (2007.). Stocks of companies that could be chosen in portfolio by investors, generally belongs to diverse industrial sectors. Global market trends, particular technologies and government activities in economy create different predis-

position for future growth of different industrial sectors. Stock markets reflect conditions of economy. Therefore, analysis of economic trends should anticipate to stocks values analysis and predictions of future values dynamics. History data analysis includes records of stock-exchange index as the consequences of the historical economic trends.

Investors are generally interested at estimation of future yields but also they need assessment of risk related to expected yields. Decision making model should consider inherent risks, as well. Investors could be less or more prone to take risky actions, and accordingly different outputs of decision making could emerged. Utility theory classified decision makers against their risk attitudes to: risk- averse, risk-neutral, and risk-seeking. Certain useful researches of empirical decision making are based on behavioural game theory and affiliating learning models. Game theory tools adjusted to investment scenarios, without players' interaction, could offer useful answers regarding the behaviour of population of rational investors which learn in laboratory conditions on their own experience [9].

Major model in our research address determination of Markowitz portfolio, and will be applied to several standard sectors at ZSE. In each sector among most liquid stocks optimal portfolio will be created. Optimal portfolios in addition will be compared. This historical analysis serves to formulate recommendations for investors as regards to orientation on industrial sectors. Simultaneously, this analysis also provides certain findings about regularities in behaviour of ZSE investors and their preferences concerning evaluation of ZSE stocks. Accordingly, optimization of some particular portfolio and precise determination of future stocks dynamics are not our prime aim. For our central model we suggested support with other appropriate analysis, mentioned earlier.

2 Industry sectors- economic policy and ZSE fundamental analysis

If regularities in behaviour of stock values are partly determined by sector's belonging than we should analyse drivers of industrial sectors. Undoubtedly, government economic policy, strategic re-orientations and restructuring activities with goals of national economy growth and development, influenced the specific industrial sectors performances.

Neoclassical approach concerning the economic growth stressed importance of free market where the state or government regulations in domain of the economic activities are undesirable. Contemporary approaches such as endogenous growth theory rejected neoclassical statements [10]. New growth theories claim that active government role in supporting advanced projects, investments in human capital and knowledge based industries, are essential for achieving long-term performance of economy [16]. In European Union specific measures toward particular industrial sectors are defined, with the goal of economic restructuring to accomplish products and industries with added-value. Since 2000 several proposals are accepted concerning the support to shipbuilding, airplane industry, and pharmaceutical industry [13]. According Porter [17] the central goal of government policy toward the economy is to deploy a nation's resources with high and rising levels of productivity. Continuous transformation of developed economies leads toward the increasing role of services. Share of services in different countries and GDP Per Capita are in high correlation (r2= 0,56) following source of World Bank [15]. In most developed countries, such are US and GB, contribution of services in added value is especially large.

Government of Croatia in recent years directed economic activities toward infrastructure projects of motorways building, administrated privatization of numerous companies, and by exchange rate policy defined climate for export and import oriented firms, as well as for tourism sector. Investors at ZSE make decisions last years based on perception of these trends and expected future trends. Unfortunately, history of trading at ZSE is not long enough to include influences of certain other events from the further history, and hence to be more reliable.

Beside the sketch of macro analysis, for the more comprehensive approach to ZSE trading the fundamental analysis is also required. Fundamental analysis of stocks respects indicators of companies earnings, assets, cash-flow, and by means of well-known ratios observed the overestimated and underestimated stocks. Among most valuable indica-

tors falls the ratio of a stock's price to its earnings per share (P/E ratio) and the ratio of market price of a share of the firm's stock divided by book value per share (P/Bv). Investors are prone to by shares of companies with lower values of P/E and P/Bv. Research conducted in 2007. On sample of different sectors at ZSE [8], showed that according to ratio P/E, sector of Transport differs significantly from the sector of Food processing and significant differences exist between sectors Banks and Industry and all other sectors. While the means of P/E for Finance (Banks) is around 19,3, for Tourism reached values of 172,5. The variable of P/Bv is a good predictor for discrimination of sectors at ZSE because there is significant difference between sectors according to analysis of variance. Sector of Transport (which has highest values- P/Bv(mean)= 5,57) differs from all other sectors. Sector of Construction (P/Bv(mean)= 3,94) differs also from Food processing sector (P/Bv(mean)= 1,84).

Research [8] indicated that in period of year 2007. At ZSE higher level of investors trust exists for the Transport and Construction companies stocks while Banks enjoy less confidence. This phenomenon partly could be explained introducing of the regulation about the criteria of investment of Croatian Retirement Funds proclaimed by the Croatian Financial Services Supervisory Agency (HANFA, CFSSA) concerning the minimal amount of market capitalization for companies. Mathematical background of our main method (Markowitz portfolio) belongs to the domain of technical analysis is exposed below. With analysis of historical data through the period of five to ten years, this method could give a suggestion concerning the optimal portfolio for different sectors at ZSE.

3 Mean and variance of portfolio of stocks - dominant portfolio in Markowitz sense

Let $S_0, S_1, \ldots, S_n$ be a record of stock prices of the stock X on trading days. Returns on the stock $x_(1,)x_(2,)\ldots, x_n$ between two periods will be calculated by the formula

$$x_i = \frac{S_i - S_{i-1}}{S_{i-1}}, \quad i = 1, \ldots, n. \tag{1}$$

Mean of the returns is equal to the arithmetical average of the historical returns

$$\mu_X = \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i. \tag{2}$$

Portfolio variance is average quadratic deviation from mean of returns, and the amount is given by formula,

$$s_X^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2. \tag{3}$$

Standard deviation of returns s_X is equal to square root of portfolio variance. Correlation measures direction and strength of linear bound between two series of data $x_1, \ldots, x_n$ i $y_1, y_2, \ldots, y_n$. The sign shows direction and amount indicates on strength of linear relationship between data series. To define correlation, we introduce covariance between data series (x_i) i (y_i) by the formula

$$s_{XY} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}), \tag{4}$$

and has to be normalized with standard deviations of data series to get the correlation factor

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}. \tag{5}$$

Factor r_{XY} is positive if both empirical series of returns (x_i) i (y_i) have tendency to have positive and negative values in the same time intervals. In our paper we used Markowitz's approach on five stocks in each sector, but we are presenting method on three stocks. For larger number of stocks $(i > 3)$ formulas can be derived similarly. Let X, Y i Z be three different stocks containing our portfolio. Let relative participation of stocks X,Y,Z, be $\omega_1, \omega_2, \omega_3 = 1 - \omega_1 - \omega_2$, respectively where $\omega_i \in [0, 1]$, $i = 1, 2, 3$. If the portfolio is rearranged after each period to keep relative participation of stocks fixed through the investment period then the return of portfolio on the end of each period equals

$$\pi_i = \omega_1 x_i + \omega_2 y_i + \omega_3 z_i, i = 1, \ldots, n. \tag{6}$$

Then return rate over the whole investing period equals

$$\mu_\pi = \omega_1 \mu_X + \omega_2 \mu_Y + \omega_3 \mu_Z \tag{7}$$

Standard deviation can't be calculated as linear combination of standard deviations of returns on stocks participating in portfolio, as it was case for the return of portfolio. Generally,

$$s_\pi \neq \omega_1 s_X + \omega_2 s_Y + \omega_3 s_Z \tag{8}$$

The same statement is worth for portfolio variance. Portfolio variance can be calculated by the use of formula

$$s_\pi^2 = \omega_1^2 s_X^2 + \omega_2^2 s_Y^2 + \omega_3^2 s_Z^2 +$$

$$+2\omega_1\omega_2 s_X s_Y r_{XY} + 2\omega_1\omega_3 s_X s_Z r_{XZ} + 2\omega_2\omega_3 s_Y s_Z r_{XZ} \tag{9}$$

Further on, since $r_{XY}, r_{XZ}, r_Y Z \leq 1$, it follows

$$s_\pi^2 \leq_1 \omega_1^2 s_X^2 + \omega_2^2 s_Y^2 + \omega_3^2 s_Z^2 +$$

$$+2\omega_1\omega_2 s_X s_Y + 2\omega_1\omega_3 s_X s_Z + 2\omega_2\omega_3 s_Y s_Z \tag{10}$$

(10) can be written in other form

$$s_\pi^2 \leq (\omega_1 s_X + \omega_2 s_Y + \omega_3 s_Z)^2 \tag{11}$$

From (11) we can conclude that standard deviation of portfolio is less or equal of the linear combination of standard deviations of stocks included in portfolio. Intuitively, since smaller standard deviation represent smaller deviation from expected return it implies smaller chance of not wonted scenarios what reduces risk. To measure success of diversification we can use coefficient of variations of portfolio. It is equal to ratio of standard deviation and rate of return.

$$CV = \frac{s_\pi}{\mu_\pi} \tag{12}$$

Coefficient of variation is proportional to standard deviation and disproportional to rate of return so we can conclude the bigger coefficient is the riskier portfolio is. If it is known expected rate of return and standard deviation of return we can build a portfolio that satisfies level of risk acceptable for individual investor. In our paper we define optimal portfolio in Markowitz sense as one that include linear combination of stocks that has minimal coefficient of variation. For finding such portfolios for each sector we have to solve problems of non-linear mathematical programming with constraints.

4 Research

For purpose of research we selected five stocks with largest trading volume in each of five observed sectors. Data are available on www.zse.hr. We were analysing following sectors and stocks construction-energetic sector (DLKV-R-A, IGH-R-A, KOEI-R-A, THNK-R-A, VDKT-R-A), transportation sector (ATPL-R-A, JNAF-R-A, LKPC-R-A, LKRI-R-A, TNPL-R-A), tourism sector (HIMR-R-A, HMAM-R-A, HUPZ-R-A, ISTT-R-A, LRH-R-A), financial sector (CROS-R-A, KABA-R-A, PBZ-R-A, RIBA-R-A, ZABA-R-A), and food sector (BLJE-R-A, LEDO-R-A, KRAS-R-A, PODR-R-A, ZAPI-R-A).

Number of stocks in observation was limited because there are not many stocks at ZSE with satisfying trading volume, specially doing the analysis by sectors. Results of research are derived from almost whole set of available stocks with trading volume criteria and surely give interesting information. Since history and stocks available for trading is relatively short due independence and privatisation of public companies trading volume became larger few years from now, and some of them don't have even long history of trading. We could recognize two significant phenomena influencing ZSE in period of Croatia becoming independent: starting of stock exchange and activation of larger and experienced investors, some of them international (2004-2006), and world economic crisis with large fall of all stock indexes through the world (2008). Each of observed sectors could have its own analysis of important events it this time period, what pass beyond research topic of this paper. One of examples would be financial sector, specially banking that had progressed significantly in period 2006.-2008., according to survey among entrepreneurs [14]. We expected appropriate evaluation of stocks in banking sector.

At Figure 1 we can see CROBEX market index value in period 2.1.2002.-1.3.2010. In our research we used monthly returns of each observed stock. Since portfolio selection has to balance return and risk we have to calculate standard deviation of returns and correlations among returns of different stocks. It has to be done while evaluating investment opportunities is not enough to ask "What is the rate of return?" , but also "Is the return sufficient to justify the risk?" [18]. After calculating average returns, standard deviations of

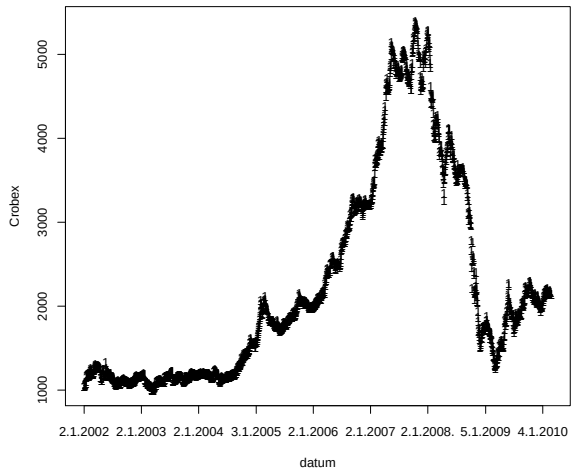

Figure 1: CROBEX market index value

returns and correlation among returns of different stock in each sector using Markowitz's approach we calculated return's and standard deviations of different portfolio possible to construct from observed stocks. Solving non-linear mathematical programming problem

$$\min s_\pi$$

under constraints $\sum_{i=1}^{5} \omega_i = 1, 0 \leq \omega_i \leq 1, i = 1 \ldots, 5$, we get portfolio with smallest standard deviation that can be recognized as the least risky portfolio. Fixing standard deviation (risk) at higher level, σ_F, then minimum calculated before we can get portfolio that maximizes return with acceptable level of risk solving non-linear programming problem maximizing $\max \mu_p i$ with additional constraint $s_\pi = \sigma_F$. Optimal portfolio in Markowitz's sense is portfolio that minimizes coefficient of variation (CV). That portfolio is calculated by solving non-linear mathematical programming problem $\min s_\pi/\mu_\pi$ under constraints $\sum_{i=1}^{5} \omega_i = 1, 0 \leq \omega_i \leq 1, i = 1, \ldots, 5$.

5 Results of research

For construction-energetic sector analysis start date is 1.8.2004. Dominant portfolio if formed as shown in Table 1. Return of optimal portfolio is $5,31\%$. Minimal coefficient of variation is $CV = 3,61$. Average correlation among returns of stocks in construction-energetic sector is $0,73$.

Table 1: Dominant portfolio for Construction-energetic sector

Stock ticker	Return	St.dv.	Opt.p.
IGH-R-A	$5,44\%$	$23,72\%$	$14,29\%$
DLKV-R-A	$3,29\%$	$19,39\%$	$0,00\%$
KOEI-R-A	$4,32\%$	$17,19\%$	$51,39\%$
VDKT-R-A	$6,75\%$	$26,94\%$	$34,33\%$
THNK-R-A	$3,37\%$	$18,67\%$	$0,00\%$

For financial sector analysis start date is 1.1.2002. Expected return of optimal portfolio is $2,095\%$, minimal coefficient of variation is $CV = 5,18$. Average correlation among returns of stocks in financial sector is $0,44$. Dominant portfolio for financial sector is formed like shown in Table 2. Only one bank isn't participating in optimal portfolio. Return of optimal portfolio is significantly smaller then for Construction-energetic sector, although time-period is larger and contains time period of significant grow of stocks at ZSE.

Table 2: Dominant portfolio for financial sector

Stock ticker	Return	St.dv.	Opt.p.
ZABA-R-A	$2,11\%$	$13,88\%$	$21,87\%$
CROS-R-A	$2,19\%$	$14,93\%$	$25,72\%$
PBZ-R-A	$2,04\%$	$12,80\%$	$30,67\%$
KABA-R-A	$1,07\%$	$11,46\%$	$0,00\%$
RIBA-R-A	$2,04\%$	$15,22\%$	$21,74\%$

For tourism sector analysis start date is 1.12.2004 (short history of significant trading with HUPZ-R-A). Dominant portfolio is formed as shown in Table 3. Return of optimal portfolio in Markowitz's sense is $2,71\%$. Minimal coefficient of variation is $CV = 3,44$. Average correlation among returns of stocks in Tourism is $0,30$.

In portfolio of transport sector are included all major shipping companies, sea ports and national company for crude oil transportation and storage of crude oil and oil products. Analysis start date is 1.12.2005 (short history of significant trading with LKPC-R-A). Dominant portfolio is formed as

Table 3: Dominant portfolio for the sector of Tourism

Stock ticker	Return	St.dv.	Opt.p.
ISTT-R-A	2,19%	11,34%	45,65%
HUPZ-R-A	3,17%	13,24%	52,83%
LRH-R-A	0,88%	15,08%	0,00%
HIMR-R-A	2,51%	21,18%	1,52%
HMAM-R-A	0,22%	12,06%	0,00%

shown in Table 4, and is formed from only two stocks. Return of optimal portfolio is 3,76% and what is relatively high amount since period of observation involves relatively short period of significant grow of ZSE. Minimal coefficient of variation is relatively high, $CV = 4,78$ since there has been significant oscillations among returns, i.e. standard deviation is high. Average correlation among returns of stocks in Tourism is quite large, and it is equal to 0,66.

Table 4: Dominant portfolio for the sector of Tourism

Stock ticker	Return	St.dv.	Opt.p.
LKPC-R-A	4,43%	22,59%	63,46%
ATPL-R-A	1,82%	18,24%	0,00%
JNAF-R-A	2,60%	18,45%	36,54%
TNPL-R-A	−0,10%	15,33%	0,00%
LKRI-R-A	2,85%	22,62%	0,00%

At Figure 2 different possible portfolios constructed from five observed stocks are shown. For example, LKPC-R-A vs. JNAF-R-A shows possible portfolios constructed only by stocks LKPC-R-A and JNAF-R-A. Optimal shows portfolio with largest return constructed under predefined standard deviation and CV line intercept those portfolios in only one point - the point that has the smallest ratio of standard deviation and return.

For selected stocks in food sector analysis start date is 1.5.2005 short history of significant trading with BLJE-R-A). Dominant portfolio in this case is constructed from only one stock Table 5. Return of dominant portfolio is equal to the stock with most significant grow, and is equal to 5,26% monthly while coefficient of variation is $CV = 3,56$. Average correlation among stocks in food sector is 0,47.

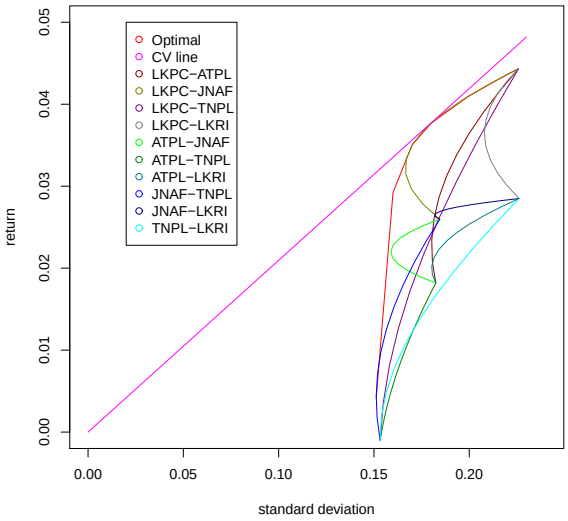

Figure 2: Optimal portfolio in Markowitz's sense for transportation sector

Table 5: Dominant portfolio for food sector

Stock ticker	Return	St.dv.	Opt.p.
PODR-R-A	0,90%	10,24%	0,00%
ZAPI-R-A	0,49%	8,04%	0,00%
LEDO-R-A	5,26%	18,73%	100,00%
KRAS-R-A	0,01%	11,66%	0,00%
BLJE-R-A	2,58%	22,19%	0,00%

Especially interesting consistency, concerning the stock sample, is found for investing in sector of Energy and Construction ($r = 0,73$) and sector of Transport ($r = 0,66$) where is recorded statistical significance, equability, in behaviour of different stocks from the same sector. Such investors' dynamics one could describe as "model of crowd". The tourism sector has the lowest correlation ($r = 0.33$).

6 Conclusions

Returns of optimal portfolios in different sectors are varying. It has to be mentioned that monthly returns in last five year have been higher then in other less risky investment opportunities. Potential investors based on presented analysis can focus their attention on construction-energetic sector where optimal portfolio have return $5,31\%$. The smallest return among chosen stocks from construction-energetic sector is $3,29\%$ (DLKV-R-A). Return on optimal portfolio in food sector is $5,26\%$, but it is presented with just one stock. For three of five stocks return less then 1%. In transport sector, return of optimal portfolio is $3,76\%$ although it has to be mentioned that only two stocks are included in the portfolio. In sector of Tourism return on Markowitz dominant portfolio is $2,71\%$. The smallest return has financial sector. It is $2,095\%$. Returns on investments at ZSE were high. Using monthly compounding with investing in optimal portfolio in financial sector we could have earned +247% on our investment in five years (without investment). Risk of investment is included in coefficient of variation. Therefore beside yields of dominated portfolios investors should comprise coefficients of portfolios as well (energy and construction- $CV = 3.61$; food industry- $CV = 3,56$; transport- $CV = 4,78$; tourism- $CV = 3,44$; financial sector- $CV = 5,18$). Results for coefficients of variation amplify recommendation for the investments in construction-energy companies. Sectors of transport and finance are overweight by enhanced risk. The major recommendation for ZSE investors, as a cumulative result of historical data analysis of prominent stocks of different industry sectors, is orientation to sector of energy and construction ($CV = 3,61$, return of dominant portfolio $5,31\%$). Additional guidelines for risk-averse decision-makers is tourism sector ($CV = 3,44$, return of dominant portfolio $2,71\%$), while for investors that accept risk attractive option could be sector of transport ($CV = 4,78$, return of dominant portfolio $3,76\%$). Appliance of Markowitz portfolio selection on the sector analysis, focusing ZSE, represents distinct contribution of this research. The last economic trends marked the decline of construction jobs and early 2010. [12], slightly lower shipping freight. Banks income are stable as well as tourism sector earnings in Croatia. There are also announcements of major projects in the field of energy industry, but also in building rail and road infrastructure [11]. In accordance with the current situation and indications of economic trends, recommendations of portfolio selection method based on analysis of historical data should be appropriately modified.

References

[1] Alexader, C., Sheedy, E. : The Professional Risk Managers' Handbook: A Comprehensive Guide to Current Theory and Best Practices, PRMIA Publications, 2005.

[2] Jorion, P., GARP: Financial Risk Manager Handbook, 4th edition, New Jersey, 2007.

[3] Munđar, D.: Optimizacija portfelja, Matematičko-fizički list (MFL), 4/236, Zagreb, 2009.

[4] RiskMetrics Group, Risk Management: A Practical Guide, 1999.

[5] Serdar,V., Šočić, I.: Uvod u statistiku, Školska knjiga, Zagreb,1986.

[6] Traiding data, available at `http://www.zse.hr`, Accessed: 10^{th} April 2010.

[7] Sarapa, N.: Teorija vjerojatnosti, Školska knjiga, Zagreb, 1992.

[8] Fabac, Robert; Dobša, Jasminka; Kadoić, Nikola: Analysis of characteristic of sectors on Zagreb stock exchange by fundamental estimators; CECIIS 2009 Conference Proceedings, Varaždin, FOI, Croatia, 2009, 203-210

[9] Fabac, Robert; Radošević, Danijel; Kozina, Melita: Empirical Learning Styles in Simulation of Investment Game, CECIIS 2008 Conference Proceedings, Varaždin, FOI, Croatia, 2008, 341-346

[10] Michael Sarel: Growth in East Asia, Economic Issues 1, Washington , D. C., 1996. International Monetary Fund

[11] `www.vlada.hr/en/content/.../file/Program\%20Vlade\%202009.-2011..pdf`, Accessed: 7^{th} May 2010.

[12] Gospodarska kretanja, br 3., Zagreb, ožujak 2010.; hgk.biznet.hr/hgk/fileovi/18220.pdf, Accessed: 7th May 2010.

[13] Teodorović Ivan, Buturac Goran: Strukturne promjene industrijske proizvodnje i komparativne prednosti Hrvatske u međunarodnoj razmjeni, Ekonomija/Economics 13(1), Pp. 221-246, 20006; www.rifin.com, Accessed: 3th May 2010.

[14] Nacionalno vijeće za konkurentnost: Godišnje izvješće o konkurentnosti Hrvatske- 2008, Zagreb, travanj 2009.; www.konkurentnost.hr, Accessed: 21th April 2010.

[15] Germany the service society - but don't forget the industrial sector!, March 6 2008; www.dbresearch.com, Accessed: 23th April 2010.

[16] North, Douglass: Institutions, Institutional Change and Economic Performance, Cambridge: Cambridge University Press., 1990

[17] Porter, Michael E.: The Competitive Advantage of Nations. New York: The Free Press, 1990. Republished with a new introduction, 1998.

[18] Higgins, Robert C.: Analysis for Financial Management, 4th ed., Richard D. Irwin Inc., 1995.