Table oriented computer software as a tool for studying dynamical systems in high school

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Abstract. *The article shows the possibility of introducing complex dynamical systems in high school, where the role of table oriented software for numerical solving differential equations is crucial. We prepared the examples with the movement of bodies under the influence of different forces. In these models we show the description of differential equations in the finite difference form with the help of table oriented software (Excel, Origin). Such introduction of complex systems enables the transfer of topics from everyday life to teaching, which reduces the gap between the theory and real experiment. By examining these problems students develop both natural science and digital competencies.*

Keywords. computer, table oriented software, mathematical modelling, dynamical systems, high school, differential and difference equations, competencies

1 Introduction

In everyday life we often deal with the natural phenomena, in physics called dynamical systems which must be described with time-dependent quantities [4]. Study of the relations between quantities usually demands finding the solution of a coupled system of differential equations. This numerical approach is a very successful scientific research method, while in the educational field it is beyond high-school students' abilities. As a consequence of the complexity of most dynamical systems and the wish of getting analytical solution of the corresponding mathematical problem, teachers deal with simplified systems too frequently, and the students learn their solutions by hearth [5]. Often the results of mathematical models do not match results of real experiments, leading to a weak understanding of dynamical systems [18].

A question arises of how the mathematical treatment of dynamical systems can be successfully transferred into education field. First of all, the visualisation of a given complex system is important [14]; we have to teach students how to decompose the problem into solvable parts and to master the composition of partial solutions into appropriate totality called the mathematical model [15, 16]. This task must be performed in close connection with experimental work. Description of mathematical relations between different quantities-variables, which are expressed as differential equations in the case of dynamical systems, is crucial. Solving differential equations in the sense of simulation of the model remains the key problem in the education field.

The ever growing use of computers in education mitigates the mentioned problem in the recent time. Computers enable mathematical treatment of complex systems in school [16, 17, 19]. At the lowest levels of education the focus is on the so called system thinking [12, 13], where the qualitative description of natural phenomena is prevailing. Later, in addition to qualitative relations between variables the mathematical description of these relations is introduced. The dependences between the variables are computed and visualized with the help of mathematical simulation.

In this article we indicate possibilities of treating complex dynamical systems in high school, where we expose the role of computer and table oriented computer software in numerical solving of differential equations. For instance, we mention the computer program Origin (`http://www.originlab.com/`) [11], which is widespread in graphic presentations of numerical calculations in scientific research, while Microsoft computer program Excel (`http://office.microsoft.com`) [10] is much preferred and well-known in school. We take an example of freely falling bodies, where the air friction is taken into account [7], in order to indicate the problem of numerical solving differential equations [1]. Furthermore, by using the program Microsoft Excel we show the possibility of introduction this physics subject in high school, what we also tested in school practice. During examining these problems students parallel develop both: natural science and digital competencies [3].

2 Example of introduction of dynamical system in education

As an example of introduction of dynamical system in education, we present movement of bodies under the influence of external forces. This problem is introduced in the study of the second Newton's low as early as in primary school. We will focus on friction force, which is usually neglected because of complex analytical solution. The simplified treatment of the problem can be misleading and cause senseless students' conclusions: 1) the speed of the falling bodies increase without limit, 2) the weight hanged on the string oscillates forever, and similar. Since the simplified solutions disagree with real experiments in these cases, the students should consider the air resistance force. This helps them to understand natural phenomena correctly. In the following, the free fall of the bodies with the air resistance will be studied. This is the movement under the influence of two external forces on the object: weight and resistance force. The 2$^{\text{nd}}$ Newton law is written as:

$$m\ddot{x} = F_g - F_R \tag{1}$$

where m is the mass of the free-falling object, F_g is the object's weight:

$$F_g = mg \tag{2}$$

and F_R is the air resistance acting on the object. The air resistance as a function of the speed is considered by the quadratic form:

$$F_R = c\frac{1}{2}\rho v^2 S \tag{3}$$

where c is the resistant coefficient, ρ is the air density, v the velocity of the free-falling objects, and S is the cross-section of the object.

The solution of eq. 1 provides the time dependence of the object's velocity and position. The solution can be expressed analytically, but it is too demanding for secondary school and not very convenient for high school. In most actual cases the systems of differential equations have no analytical solutions and require numerical methods, such as Runge-Kuta [1]. In dealing with these calculations, we use various programme languages like C++, Pascal or Basic. Recently, equation-oriented programmes appear more and more frequently (Auto [6], Berkeley Madonna [8]): they contain built-in different numerical methods. Of course, this kind of work is suitable in scientific area and for high-school students. It is not acceptable at lower educational levels due to the lack of knowledge in mathematics, programming and numerical methods. We will demonstrate the possibility of numerical solving differential equations with the finite difference method in high school by using Microsoft Excel.

3 Solving Finite-Difference Equations

The acceleration in Eq. 1 gives the rate of the velocity change with time, $\frac{dv}{dt}$, and the velocity in turn determines the rate of the position change, $\frac{dx}{dt}$, thus the corresponding pair of differential I-order equations is:

$$\dot{x} = v \tag{4}$$

$$\dot{v} = g(1 - \frac{v^2}{v_0^2}) \tag{5}$$

where v_0 is the terminal speed that can be achieved by the body: $v_0 = \sqrt{\frac{2mg}{cS\rho}}$. In order to solve the system of model equations (eqs. 4-5) numerically, a simple finite-difference formalism can be used. Here we would like to point out that complex schemes, such as Runge-Kutta, are not always needed. For educational purpose very simple schemes are satisfactory, e.g.:

$$x(t + \Delta t) = x(t) + \Delta t \cdot v \tag{6}$$

$$v(t + \Delta t) = v(t) + \Delta t \cdot a, \text{ where} \tag{7}$$

$$a = g(1 - \frac{v(t)^2}{v_0^2}) \tag{8}$$

. The system of differential equations (eqs. 4-5) is translated into the system of algebraic equations (eqs. 6-7), which can be implemented in any programme package that supports table-calculations (e.g. Origin or Microsoft Excel). For the implementation in Excel see Fig. 1.

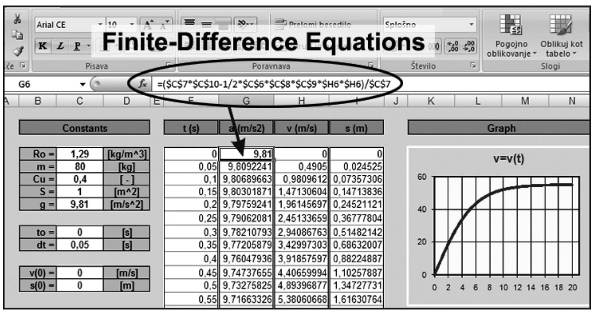

Figure 1: Implementation of the model (eqs. 6-7) in Microsoft Excel.

On the left side of Fig. 1 constants present in Eq. 7 are shown. They can be varied to study their influence on time evolution of the object's position. In the middle part of the document the tabellary calculations of the variables a, v and x are presented (the time interval $\Delta t = 0.05$ s was chosen). On the right, the diagram of the time dependence of velocity is shown.

The importance of finite-difference equations for introducing mathematical modelling in school is appreciated well. Examples of mathematical modelling of physical problems by using the finite-difference equations are already included in some textbooks for physics [9, 2].

4 Testing in school

The example of the free-falling body with air resistance was tested in school, in the third and fourth year of gymnasium. The physical problem was presented to students, and then they solved it numerically as described above. The teacher leaded the students through individual steps of the procedure, evaluating their successfulness by numbers from 1 to 5 (1 = the student did not succeed to adopt some step, 5 = the student performed the step very successfully). The averages of the results for individual steps are shown in Fig. 2.

2 indicates, that recording the 2^{nd} Newton law (step a) presented the least trouble to students, since they were fairly familiar with the case. Some small difficulties appeared in the recording the force of air resistance (eq. 3). Translation of Eq. 1 into the system of two equations (eqs. 4-5, step b) proved to be much more tricky, where the most difficult part was the introduction of the terminal speed v_0. Comprehension of the symbolism of equations was also slightly problematic, particularly the meaning of $a = \frac{dv}{dt}$ and $v = \frac{dx}{dt}$. There were smaller difficulties in the next step of writing the equations in difference form (step c) in comparison to step b, because the corresponding procedure have already been presented to students in the case of similar problem. Similarly, the last two steps (d and e), meaning the work with computer, presented no serious problems since the students were familiar with the programme Excel. There were just a few standard problems with fixing individual cells for input of constants that should be used in "working" vari-

able cells in middle columns. Graphic presentation was easy for students.

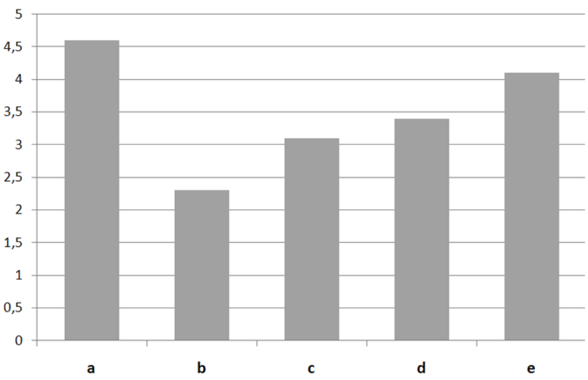

Figure 2: Average marks for all students for individual steps of the computation/presentation procedure. a) Record of the II. Newton law (eqs. 1-3). b) Record of differential equations (eqs. 4-5). c) Translation to finite difference equations (eqs. 6-7). d) Inserting finite difference equations into Excel document. e) Graphic presentation of results.

5 Discussion

The problem of solving differential equations appearing in the mathematical treatment of dynamical systems was exposed in this paper. Since fulfilling this task directly is too demanding for students in the secondary school, the alternative is searched. We have used an example of the falling body in the air to indicate the possibility of numerical solution of the problem. Tabellary oriented computer programmes can be of great help, because they enable the fast computation of numerical simulations. The generally known and widespread programme Excel was used for our purpose. It was shown that this programme offers good possibilities for numerical solving differential equations in finite difference form, and that students have no big difficulties in using this familiar software tool.

This method can significantly improve the students' understanding of several physics problems. Many of these problems can be stated with a larger degree of realism while at the same time the students are trained to problem solving strategies. In this way the student not solely understands the considered natural phenomenon but becomes able to transfer his/her knowledge (skill) to other dynamical processes, too. For example, the studied problem of falling bodies could be generalized to the kinematics of the objects in different media and in the geometries with different dimensionalities (e.g., inclined throw in the air). This case is relevant in different sports where we deal with rapid movement of light objects, such as badminton ball. We suppose that many disciplines would appear to be more interesting, attractive and useful to students by more frequent application of the described numerical method and the software tools. In addition, by examining these problems students simultaneously develop their natural science and digital competencies [3].

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