The Results of Comparison Study of Some Algorithms of Simulation Optimization

Pavel Važan, Pavol Tanuška, Gabriela Križanová
Faculty of Material Science and Technology in Trnava
Slovak University of Technology in Bratislava
Paulinska 16, 917 24 Trnava, Slovakia
{pavel.vazan, pavol.tanuska, gabriela.krizanova}@stuba.sk

Abstract*. The paper is a contribution to more effective using of simulation optimization. The article presents the results of the comparison study of selected algorithms of simulation optimization. The authors have tested several algorithms on various simulation models. The simulation models of systems were created in simulator Witness. The main goal was to give the general procedure for effective usage of simulation optimization. The authors point out that the realization of simulation optimization is a compromise between acceptable time and accuracy of found solution. The final procedure involves the process of selection of algorithm, input variables, their set up of range and step selection. The proposal of the authors has already been realized and has brought significant time reduction.*

Keywords. Simulation, optimization, model, algorithm

1 Introduction

The simulation optimization is the most significant simulation technology in the last years according to many authors. It eliminates one of disadvantages of simulation and it is used to find the best solution from many simulation experiments.

The combination of simulation and optimization has already been expected for a long time, but only in the last decade it has achieved real development.

Today, leading simulation software vendors have introduced optimizers that are fully integrated into their simulation packages. Now simulation practitioners have access to robust optimization algorithms and they are using them to solve a variety of "real world" simulation optimization problems [5].

There also exist many barriers which have to be over-came for broader simulation optimization using. Great scepticism predominates to the results of simulation optimization in concrete applications [2].

2 The methods of simulation optimization

Understandably there are a lot of methods that could be used for simulation optimization. The major simulation optimization methods are displayed on Fig.1 [1]. However, most developers have involved heuristic search methods into the software packages for simulation optimization. The heuristic search algorithms provide good, reasonably fast results on a wide variety of problems [1].

We want to mention a few important heuristic algorithms. Here belong genetics algorithms, evolutionary strategies, simulated annealing, simplex search, tabu search [4].

The computational demands of simulation optimization cause, that the practical usage of simulation optimization is possible without software support. The software packages are solved as plug-in modules which are added to the basic simulation platform. The approach to simulation optimization is based on viewing of the simulation model as a black box function evaluator. The optimizer chooses a set of values for the input parameters and uses the responses generated by the simulation model to make decisions regarding the selection of the next trial solution.

The software available today does not guarantee that it locates the optimal solution in the shortest amount of time for all possible problems that it may encounter. That would be a monumental accomplishment. However, the target was to develop and provide algorithms that could consistently find good solutions to problems that are better than the solutions analysts were finding on their own (manually). It is evident that the current software has demonstrated its usefulness.

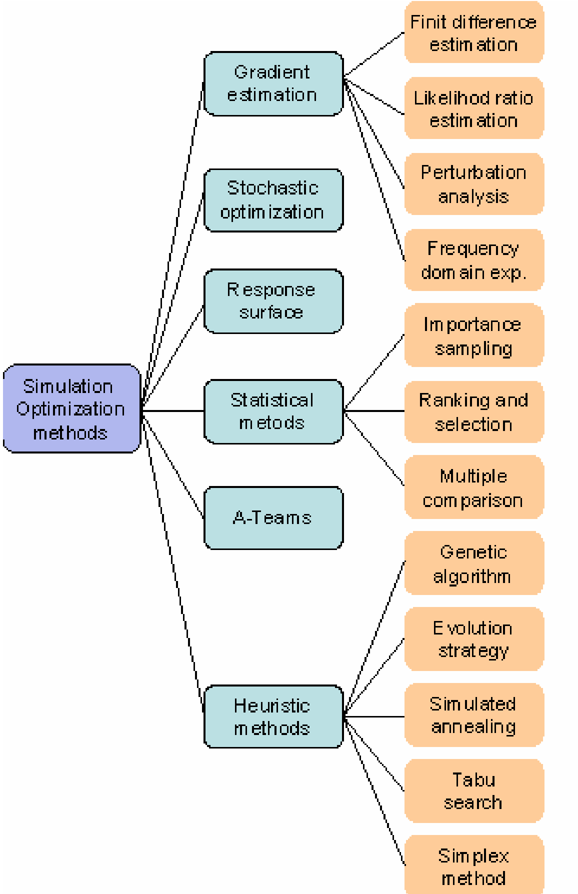

Figure 1. The methods of simulation optimization

3 Comparison of selected Algorithms

In spite of the progresses it is necessary to emphasize that the realization of simulation optimization will always be a compromise between acceptable time and accuracy of found solution.

The authors realized the research oriented to comparison of selected algorithms of simulation optimization. The following criteria were compared:

- Time for optimal solution finding
- Success of optimal value finding
- Number of runs for evaluation
- The influence of parameters change of algorithms for optimal solution finding

The authors have tested several algorithms on four different simulation models. Three simulation models represented the manufacturing systems and one simulation model was the model of the business process oriented system. Two models were stochastic and two models were deterministic systems.

The main goal was to evaluate selected algorithms not only from the point of view of accuracy of found global extreme of objective function but also from the point of view of time which was necessary for searching of the global extreme.

The authors have searched such sequence of steps that the selected algorithm will find the best result in the shortest time.

The process of simulation optimization is limited by many factors. Here belong [3]:

- the number of possible combination;
- constrains – The definition of constrains can dramatically decreased the number of combinations. Constrains have to be defined on the base of knowledge about the simulated system;
- the number of input variables - This number has considerable influence on accuracy and the time of the optimum searching;
- the range of input variables – The limitation of range of variables is very important part of simulation optimization;
- the number of runs that are needed to obtain one value of objective function. It is typical for stochastic simulation models;
- time of simulation run;
- warm up period.

The limited range of this paper does not allow present the whole extensive work. This is why we show the problem of successful usage of simulation optimization only on the chosen example. This process was used for searching of optimal values of lot size in the real production system.

The simulation model of system was created in simulator Witness. See fig. 2.

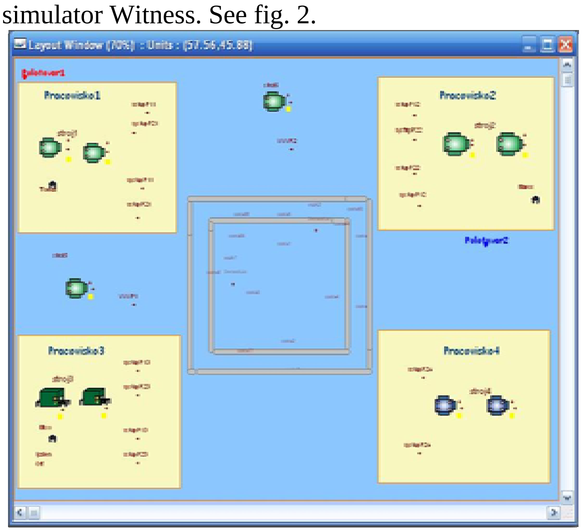
Figure 2. Simulation model of FMS

Flexible manufacturing system consisted of four machine groups. There were two relative kinds of products named VD1 and VD2 in manufacturing system produced at the same time. The lot sizes were set up to 5 pieces. The schedule of operations was created for every type of product. The sequence of operations was created for every machine group but the realization of operation on the concrete machine in the group was decided by immediate situation.

The basic advantage of simulation optimization is that the objective function can be defined in a simple way and it does not need to contain input values. The objective function is defined inside the simulation model.

The procedure was defined as follows [4].:

IF No_out_parts () < default value of finished parts AND Machine utilisation () < default value of machine utilisation AND Flow time () > default value of flow time

Unit_Costs = SumCosts / No_out_parts

RETURN Unit_Costs

ELSE

Unit_Costs = SumCosts / No_out_parts+ constant1

RETURN Unit_Costs

ENDIF

Objective function returns the value of costs per finished part when quantitative values of defined manufacturing goals are fulfilled. At the beginning it is necessary to define default values of flow time, machine utilization and number of finished parts. These values were found out by simulation way from so called preparatory experiments. And these preparatory experiments were specially designed for this purpose:

- default value of finished parts – 500 parts
- default value of machine utilisation – 75%
- default value of flow time – 150 minutes
- constant1 - 1000

The realization of simulation optimization has been realized in plug-in module Optimizer.

These algorithms were included into the comparison:

- All combinations – brute force algorithm.
- Random solutions – to enable an appreciation of the shape of the solution space
- Min/Mid/Max – tests the extremes and mid points of all parameter settings. Covers all options for non-range parameters.
- Hill Climb – a simple algorithm. Fast but prone to get stuck in local optima.
- Adaptive Thermostatistical simulated annealing – the main algorithm, a variant of simulated annealing with extra adaptive nature. Developed by Lanner.

Understandably the authors have respected individual parameters of selected algorithms. These were mainly maximum evaluations and maximum moves without improvement with the exception of algorithms All combinations and Min/Mid/Max. These algorithms have no individual settings. The rest of algorithms had set maximum evaluations = 800 and maximum moves without improvement =300.

The simulation model has 4 input variables - lot sizes for both products (LS1, LS2 in tables) and input intervals for both products (P1, P2 in tables). The right values of input intervals of the lot sizes also have to be connected because lot size and their input interval hang together. It is very important to constrain the input parameters meaningfully. Accurate setting up of the input intervals markedly reduces the optimization time.

3.1 Experiments

The number of optimization variables and their ranges is defined by the following table 1.

Table 1. The number and range of input variables.

Var. name	Range of var.s	No values	No combinations
P1	5 - 30	26	26
P2	5 - 70	66	1716
LS1	1 - 10	10	17160
LS2	1 -10	10	171600

The total number of possible combination is too big. We are searching for the minimum of objective function (minimal value of total costs). The simulation run takes 7280 minutes (5 working days, 12 hours shift), warm up period lasts 80 minutes.

Table 2. Results of experiment1.

Algorithm	Found optimum	Estimated time	Real time	No evaluations
Adaptive T.SA	202	0:16:23	0:05:15	658
All Comb.	----------	2days23h	-------	-----------
Random Solutions	216	0:17:29	0:17:51	800
Min/Mid/Max	271	0:01:48	0:01:48	81
Hill Climb	239	0:18:24	0:00:26	312

It is not possible to use the algorithm All combination because of long time. This follows from the results. The other algorithms have found only approximate values of optimum. See table 2.

In the next experiments we have tried to reduce scanned set and then bigger step 3 has been chosen for variables P1 and P2 and step 2 for variables LS1 and LS2. The number of possible combinations has decreased to 8280. The results of these experiments are presented in table 3:

Table 3. Results of experiment2

Algorithms	Found optimu m	Estimated time	Real time	No evaluations
Adaptive T.SA	189	0:27:51	0:06:59	627
All Combin.	189	2:58:17	2:56:48	8280
Random Solutions	249	0:24:16	0:23:56	800
Min/Mid/Max	271	0:01:50	0:01:40	81
Hill Climb	252	0:25:00	0:00:36	317

The dramatic reduction of the number of combinations allowed use algorithm All combinations as the results of experiment show. Then the better value of objective function has been found in previous experiment. The found value need not to be the searched minimum. This has been caused by the reduction of the number of combinations.

However, better value that has been found allows reduce the range of input variables. Here is important to remark that when the global extreme was found on the border of input variable range then it is necessary to repeat the experiment. In this case it is needed to increase the range of input variables. The searched optimum has to be inside of the definition scope of input variables.

If the range of input variables is reduced enough, it will be possible to use step 1. The new ranges of variables are presented in table 4 in this case.

Table 4. New ranges of variables

Var. name	Old range of var.	New range of var.
P1	5 - 30	8 - 14
P2	5 - 70	7 - 13
LS1	1 - 10	1 - 3
LS2	1 - 10	1 - 3

If the number of combinations allows use algorithm All combinations then it will be possible to find the global extreme of the objective function in relatively short time. It is possible to declare that the solution is really searched global minimum of objective function. The table 5 documents the last step of this procedure.

Table 5. Final results

Algorithm	Found optimum	Estimated time	Real time	No evaluations
Adaptive T.SA	162	0:18:09	0:01:34	321
All Comb.	155	0:25:00	0:22:28	990
Random Solutions	155	0:17:48	0:12:39	800
M/M/M	170	0:01:49	0:01:48	81
Hill Climb	1114	0:27:07	0:00:23	311

The found value 155 is real minimum of objective function. This value was reached by algorithm All combination and also algorithm Random solution but their time was too long. Algorithm Hill climb probably get stuck in local extreme.

We have also experimented with individual internal parameters of algorithms. It was especially the maximum evaluations and the maximum moves without improvement. It is not possible to set up these parameters for algorithms All combination and Min/Mid/Max. We have tested specific situation. We reduced total number of possible combinations of input variables on 441. Then we set up the maximum moves without improvement to 441 for all algorithms (understandably without algorithms All combination and Min/Mid/Max). We can note that all algorithms had the same condition in this case. The table 6 presents the results of this special experiment.

Table 6: The results of the special experiment

Algorithm	Found optimum	Estimated time	Real time	No evaluations
Adaptive T.SA	162	0:15:15	0:14:31	441
All Comb.	162	0:14:23	0:15:06	441
Random Solutions	162	0:14:30	0:14:05	441
M/M/M	162	0:02:35	0:02:34	81
Hill Climb	163	0:25:26	0:00:37	441

All algorithms with the exception of Hill climb algorithm found the same value, but their time of search was different. This example is not typical situation of simulation optimization solution. Usually the number of permissible combination of input variables is much bigger. We recommend usage All combination algorithm for less than 1000 combinations of input variables for majority of simulation models. We suggest set up the maximum evaluations to 800 and the maximum moves without improvement to 300 according to results of our experiments.

4 The evaluation of tested algorithms

Although simulation optimization represents powerful tool, its real usage needs complex knowledge and appropriate procedure. The selection of the proper algorithm is the important part of this procedure.

The brute force algorithm All combinations always guarantees optimal value finding, but the time of optimum searching is too long and unacceptable in many cases. Usually it is impossible to use this algorithm at the beginning of optimization process.

The algorithm Adaptive Thermostatistical Simulated Annealing was the best in majority of experiments from the point of view of accuracy and the searching time. This algorithm had very good results also at bigger searching step. We recommend usage of Adaptive Thermostatistical Simulated Annealing algorithm if the total number of input values combination is great and also if the searching step is bigger.

The algorithm Random Solutions chooses random values from set of variables. It has reached good results when the total number of input values combination was big and the searching step was 1. It is advantegous to use the algorithm Random Solutions at the beginning of optimization process. It offers overview of scanned space. Then it is possible to reduce this space according to the results of the algorithm.

The algorithm Min/Mid/Max have tested only minimal, middle and maximal values of input variables, therefore the number of tested combinations was relatively small. The short searching time was up to the small sample, but the accuracy was low. The efficiency of the algorithm Min/Mid/Max is better for narrow ranges of input variables.

The algorithm Hill Climb was the fastest from all tested algorithms. It has to get on local extreme very often, therefore its results were the worst. It is possible to use this algorithm in the final step of optimization when the ranges of input variables are narrow and only one extreme is expected.

4.1 The next factors which influence simulation optimization

The next factors which influence simulation optimization are:

- the length of simulation runs. It directly influences the time of simulation optimization realization.

- the input variables and their ranges. The appropriate definition of input variables is the one of the most important steps of simulation optimization. The selection of variables that influences the objective function has to follow from knowledge of solved problem and its representation in simulation model. Understandably the number of input variable and their ranges markedly influence the time of simulation optimization process. We recommend verification of ranges of input variables by specially designed preparatory experiments. The goal of these experiments is very sensitively set up the range of variables so that the total number of possible combinations will be minimal.

- the search step. It influences the quality of optimization. The search step should be defined on dependence on the number and ranges of input variables. We suggest set up the search step 2 or 3 for wide range (30 and more values). The bigger step is usually defined for the first optimization experiment. Here exist two criteria for search step determination – the reduction of the number of combination and the reduction of time of optimization. The determination of search step is related to algorithm selection.

- the number of simulation runs for the objective function evaluation. It directly influences the time of optimization process. The number of simulation runs depends on the character of simulation model. It is necessary to realized more simulation runs when the model is stochastic. Only one simulation run is needed for deterministic model.

5 The proposal of final procedure

The authors recommend the following procedure (simplified) for algorithm selection and realization of optimization process based on extensive research:

To reduce the range of inputs variables by specially designed preparing experiments. The right range represents such states of the system that will be explored. The constraints of inputs variables represent upper and lower limits for system loading in the presented example.

Use algorithm Adaptive Thermostatistical SA with searching step 1 for model with many input variables but their range is small. The value of searching extreme is very near to optimum however the searching time is very often shorter than when used with the algorithm All combinations.

We suggest the following procedure for models with few input variables (less than 5) but their range is very huge (permissible combination can be from several hundred thousands to several million):

1. Use algorithm Random solutions.
2. Use Adaptive Thermostatistical SA with bigger step (2 and more).
3. Compare both results.
4. Reduce the range of input variables on the base of the best result of both algorithms.
5. Repeat experiment by using the Adaptive Thermostatistical SA algorithm with searching step1.
6. Reduce again the range of input variables and repeat experiment by using the Adaptive Thermostatistical SA algorithm. (Important note: When the extreme is found on the border of input variable range then it is necessary to repeat the experiment.)
7. If it is possible to reduce the range of input parameters again or if time of obtained result is acceptable, then repeat the experiment by using All combinations algorithm or Hill Climb algorithm, else repeat the experiment by using the Adaptive Thermostatistical Simulated Annealing algorithm.

6 Conclusion

There are more areas where simulation optimization can be used. Of course the choice of the procedure used in simulation optimization depends on the analyst and the solved problem. Our research made a contribution to practical usage of simulation optimization. The final procedure allows reduce the time of global extreme searching but the accuracy of finding solution is very high. Usually we have found the global extreme of the objective function precisely. We believe that the increasing of the efficiency and simplicity of applications of simulation optimization can be valuable.

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References

[1] April, J.; Glover, F.; Kelly, J.P. & Laguna M. **Practical introduction to simulation optimization**, Proceedings of the WSC 2003., Chick, P. J. Sánchez, D. (Eds.), pp. 71-77, ISBN: 0-7803-8131-9 New Orleans, 12. 2003, WSC.

[2] Boesel J., Glover F., Bowden O.R., Kelly J. P. & Westvig E. (2001). **Future of simulation optimization.** Proceedings of the 2001 Winter Simulation Conference, pp. 1466-1469, ISBN 0-7803-7309-X 12,2001, ACM, New York.

[3] Bc. Mrva, M: **The comparison of selected algorithms of simulation optimization**. [Diploma thesis] – Slovak University of Technology in Bratislava. Faculty of Materials Science and Technology (MtF STU); – Diploma thesis tutor: Doc. Ing. Pavel Važan, PhD. – Trnava: MtF STU, 2008, 111 pages.

[4] Važan, P., Schreiber, P. & Križanová G. **The Simulation Optimization in Simulator Witness**. Proceedings of the Third Conference: Ecumict pp. 461-469, ISBN 9-78908082-553-6 Gent, 3, 2008. Nevelland

[5] Waller A.P. **Optimization of simulation experiments**, Available from : http://www.dynamic.co.kr/Witness_Training_Center/Articles/White%20paper%20-%20Optimization.pdf Accessed: 2009-04-28