

Towards Memory-Efficient Topological Feature Extraction for Biomedical Signals on Resource-Constrained Platforms

Prema memorijski učinkovitom izdvajanju topoloških značajki za biomedicinske signale na platformama s ograničenim resursima

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Abstract. Edge-intelligence systems face a computational trilemma of precision, latency, and energy consumption. This paper introduces a hardware-software co-design framework utilizing Persistent Sheaf Laplacians for geometry-aware feature extraction. We present an optimized C/C++ Vietoris-Rips implementation that flattens symmetric distance matrices to reduce SRAM usage by 50%, alongside an Early-Terminated Union-Find algorithm halting topological reductions at the sheaf global section threshold. Simulations using MIT-BIH and PPG-DaLiA datasets demonstrate execution latencies under 8 ms and a 31.75 KB peak memory footprint, establishing real-time feasibility for embedded medical wearables

Keywords. Topološka analiza podataka, perzistentni snopovni Laplaceov operator, kibernetičko-fizički sustavi, zajednički dizajn sklopovlja i programske opreme, detekcija anomalija

Sažetak. Sustavi rubne inteligencije suočavaju se s trilemom preciznosti, latencije i energije, dok je klasična perzistentna homologija ograničena DRAM pristupu. Ovaj rad predlaže okvir zajedničkog dizajna sklopovlja i programske opreme s perzistentnim snopovnim Laplaceovim operatorima za lokalizirano izdvajanje značajki. Uvodimo optimiziranu C/C++ implementaciju Vietoris-Ripsovog kompleksa s izravnavanjem matrice u 1D niz, smanjujući SRAM za 50%. Rano prekinuti Union-Find algoritam zaustavlja redukcije kod β_0 . Profiliranje u ARM Cortex-M okruženju uz MIT-BIH i PPG-DaLiA pokazuje latencije ispod 8 ms i memorijski otisak od 31,75 KB za 128 uzoraka, potvrdivši izvedivost detekcije anomalija u nosivim uređajima.

Ključne riječi. Topological Data Analysis, Persistent Sheaf Laplacian, Cyber-Physical Systems, Hardware-Software Co-Design, Anomaly Detection

1 Introduction

Raw-data streaming in edge-intelligence systems - such as wearable physiological monitors - faces a severe "computational trilemma" involving precision, latency, and energy consumption. The primary bottleneck for edge devices is not arithmetic computation, but rather the energy cost of off-chip memory access and external data transmission. Continuously streaming high-dimensional raw time-

1 Uvod

Prijenos sirovih podataka u sustavima rubne inteligencije - kao što su nosivi fiziološki monitori - suočava se s ozbiljnom „računalnom trilemom” koja uključuje preciznost, latenciju i potrošnju energije. Primarno usko grlo za rubne uređaje nije aritmetičko računanje, već energetska trošak pristupa izvančipnoj memoriji i vanjskog prijenosa podataka. Kontinuirani prijenos visokodimenzionalnih sirovih podataka vremenskih nizova u oblak troši za nekoliko redova

series data to the cloud consumes orders of magnitude more energy than processing data locally on-chip, rendering cloud-dependent streaming unsustainable for the stringent power envelopes of embedded medical wearables (Warden and Situnayake, 2019; Ray, 2022).

Deploying topological data analysis (TDA) (Carlsson, 2009; Edelsbrunner and Harer, 2008) at the extreme edge introduces significant engineering trade-offs. For a time series of length N , constructing a classical Vietoris-Rips complex requires evaluating all pairwise metric distances, scaling at a minimum quadratic complexity of $O(N^2)$. Furthermore, conventional persistent homology suffers from combinatorial explosion during boundary matrix reduction, which typically requires worst-case cubic time complexity, $O(M^3)$ (Otter et al., 2017; Bois et al., 2025). Even state-of-the-art computational optimizations (Bauer, 2021) clash directly with the constraints of microcontroller units (MCUs), which typically feature less than one megabyte of on-chip Static Random-Access Memory (SRAM).

To resolve this computational trilemma, this paper utilizes a cellular sheaf-theoretic representation to shift the paradigm from raw data transmission to highly localized, geometry-aware feature extraction.

velečine više energije nego lokalna obrada podataka na čipu, što prijenos ovisan o oblaku čini neodrživim za stroge energetske okvire ugrađenih medicinskih nosivih uređaja (Warden i Situnayake, 2019.; Ray, 2022.).

Implementacija topološke analize podataka (TDA) (Carlsson, 2009.; Edelsbrunner i Harer, 2008.) na samom rubu donosi značajne inženjerske kompromise. Za vremenski niz duljine N , konstrukcija klasičnog Vietoris-Ripsovog kompleksa zahtijeva evaluaciju svih parnih metričkih udaljenosti, skalirajući se s minimalno kvadratnom složenosti od $O(N^2)$. Nadalje, konvencionalna perzistentna homologija pati od kombinatorne eksplozije tijekom redukcije granične matrice, što obično zahtijeva kubičnu vremensku složenost u najgorem slučaju, $O(M^3)$ (Otter et al., 2017.; Bois et al., 2025.). Čak i najsuvremenije računalne optimizacije (Bauer, 2021.) izravno se sukobljavaju s ograničenjima mikrokontrolerskih jedinica (MCU), koje obično imaju manje od jednog megabajta statičke memorije s nasumičnim pristupom na čipu (SRAM).

Kako bi se riješila ova računalna trilema, ovaj rad koristi teorijski prikaz celularnih snopova kako bi se paradigma pomaknula s prijenosa sirovih podataka na visoko lokalizirano izdvajanje značajki svjesno geometrije.

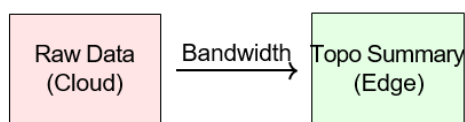
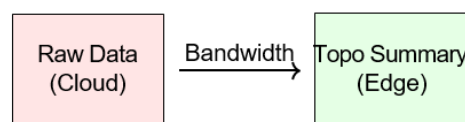


Figure 1. The paradigm shift: Local topological feature extraction avoids the energy-intensive cloud streaming of raw physiological data.



Slika 1. Promjena paradigme: Lokalno izdvajanje topoloških značajki izbjegava energetski intenzivno strujanje sirovih fizioloških podataka u oblak.

Summary of Contributions:

- **Algorithmic Compression:** We propose a 1D symmetric distance matrix flattening technique that reduces the memory requirements of topological graph construction by over 50%, preventing dynamic SRAM overflow.
- **Computational Acceleration:** We introduce an Early-Terminated Union-Find algorithm that halts boundary reduction once the sheaf global section is reached, enabling execution latencies securely under 8 ms.
- **Preliminary Signal-Level Evaluation:** We validate the framework on the MIT-BIH and PPG-DaLiA datasets, suggests computational feasibility that localized sheaf features successfully isolate physiological anomalies from heterophilic motion artifacts.

Sažetak doprinosa:

- **Algoritamska kompresija:** Predlažemo tehniku izravnjavanja simetrične matrice udaljenosti u 1D niz koja smanjuje memorijske zahtjeve konstrukcije topološkog grafa za više od 50%, sprječavajući dinamičko prelijevanje SRAM-a.
- **Računalno ubrzanje:** Uvodimo rano prekinuti Union-Find algoritam koji zaustavlja redukciju granica kada se dosegne globalni presjek snopa, omogućujući latencije izvršavanja koje su sigurno ispod 8 ms.
- **Preliminarna evaluacija na razini signala:** Validiramo okvir na skupovima podataka MIT-BIH i PPG-DaLiA, što ukazuje na računalnu izvedivost toga da lokalizirane značajke snopa uspješno izoliraju fiziološke anomalije od heterofilnih artefakata kretanja.

2 Related work

2.1 Limitations of Standard Graph Networks

The development of Graph Neural Networks (GNNs) has indicated a standard paradigm for modeling relational data. However, conventional GNNs impose severe performance bottlenecks when applied to heterophilic networks and non-stationary physiological time-series (Ju et al., 2024). Mainstream GNN architectures operate under the assumption of homophily - the premise that connected nodes share identical features. Spatial neighborhood aggregation recursively averages feature representations, behaving as a low-pass filter that smooths out local high-frequency variations. When applied to physiological systems characterized by sudden anomalies, this low-pass filter destroys critical localized signatures. Recent works have introduced Cellular Sheaves and Neural Sheaf Diffusion (Bodnar et al., 2021; Hansen et al., 2020) as a geometric mechanism to overcome this oversmoothing, allowing models to act as geometry-aware high-pass filters.

Table 1. Comparison of Relational Modeling Paradigms

Paradigm	Heterophily Handling	Edge Hardware Feasibility
Standard GNNs	Poor (Oversmoothing)	Moderate
Classical TDA	Excellent	Low (SRAM Overflow)
Persistent Sheaf (Ours)	Excellent (Restriction Maps)	High (1D Compression)

2.2 Topological Data Analysis on Edge Hardware

TDA has proven highly effective in extracting structural features from complex molecular and physical systems (Larraz, 2026; Wang et al., 2025). Recent advances in edge computing have introduced energy-aware machine learning models, such as MCUNet (Lin et al., 2020), which optimize neural architectures for microcontrollers. Concurrently, efforts to optimize data transmission on resource-scarce hardware have explored advanced edge-computing mechanisms, such as real-time D-PMU data compression for digital distribution networks (Yang et al., 2023). However, deploying Persistent

2 Srodni radovi

2.1 Ograničenja standardnih grafovskih mreža

Razvoj grafovskih neuronskih mreža (GNN) ukazao je na standardnu paradigmu za modeliranje relacijskih podataka. Međutim, konvencionalni GNN-ovi nameću ozbiljna uska grla u performansama kada se primijene na heterofilne mreže i nestacionarne fiziološke vremenske nizove (Ju et al., 2024.). Vodeće GNN arhitekture djeluju pod pretpostavkom $\{homofilije\}$ — premise da povezani čvorovi dijele identične značajke. Prostorna agregacija susjedstva rekurzivno usrednjava prikaze značajki, ponašajući se kao niskopropusni filter koji izgladuje lokalne visokofrekventne varijacije. Kada se primijeni na fiziološke sustave koje karakteriziraju iznenadne anomalije, ovaj niskopropusni filter uništava ključne lokalizirane potpise. Nedavni radovi uveli su celularne snopove i neuronsku difuziju snopova (Bodnar et al., 2021.; Hansen et al., 2020.) kao geometrijski mehanizam za prevladavanje ovog prekomjernog izgladivanja, omogućujući modelima da djeluju kao visokopropusni filteri svjesni geometrije.

Tablica 1. Usporedba paradigma relacijskog modeliranja

Paradigm	Heterophily Handling	Edge Hardware Feasibility
Standardne GNN mreže	Slabo (Prekomjerno izgladivanje)	Umjereno
Klasična TDA	Izvršno	Nisko (Prelijevanje SRAM-a)
Perzistentni snop (Naš)	Izvršno (Mape restrikcije)	Visoko (1D kompresija)

2.2 Topološka analiza podataka na rubnom sklopovlju

TDA se pokazala iznimno učinkovitom u izdvajanju strukturnih značajki iz složenih molekularnih i fizičkih sustava (Larraz, 2026.; Wang et al., 2025.). Nedavni napredak u rubnom računarstvu uveo je modele strojnog učenja svjesne potrošnje energije, kao što je MCUNet (Lin et al., 2020.), koji optimiziraju neuronske arhitekture za mikrokontrolere. Istodobno, naporima za optimizaciju prijenosa podataka na sklopovlju s ograničenim resursima istražili su napredne mehanizme rubnog računarstva, kao što je kompresija D-PMU podataka u stvarnom vremenu za digitalne distribucijske mreže

topological Laplacians (Wei et al., 2025) on resource-scarce hardware still requires radical algorithmic compression to avoid SRAM overflow, a gap this paper directly addresses (Table 1).

3 Theoretical Framework

Sheaf theory facilitates distributed signal fusion by capturing multi-modal data streams directly on the device. Persistent sheaf Laplacians encode both geometrical and non-geometrical information (Wei et al., 2025), capturing multiscale flexibility in biomolecular systems (Hayes et al, 2025). We extend this property to non-stationary cardiovascular signals.

- Local Opinions and Restriction Maps: Cellular sheaf theory equips a sensor network with a geometric structure. The raw signal captured by a sensor node acts as its "private opinion"
- The Coboundary Operator: The sheaf Laplacian explicitly measures the aggregated "disagreement of opinions" at each sensor node. It is formulated using block-matrix coboundary operators δ_h .

$$L_h = \delta_h^T \delta_h + \delta_{h-1} \delta_{h-1}^T \quad (1)$$

Normal physiological states are modeled to reside within the Harmonic Space ($H^0(G,F) = \ker(L_0)$), representing global consensus. Conversely, sudden anomalies (arrhythmias or motion artifacts) are projected outside this kernel as topological disagreements. Because this computation is driven by restriction maps linking adjacent nodes, the operations are transformed into sparse linear algebra routines that fit comfortably within constrained on-chip SRAM.

4 Hardware-Software Co-Design

To bypass the memory overhead of Python libraries, we implement a bare-metal C/C++ architecture tailored for Cortex-M processors. The proposed architecture is illustrated in Fig. 2.

(Yang et al., 2023.). Međutim, implementacija perzistentnih topoloških Laplaceovih operatora (Wei et al., 2025.) na sklopovlju s ograničenim resursima i dalje zahtijeva radikalnu algoritamsku kompresiju kako bi se izbjeglo prelijevanje SRAM-a, što je jaz koji ovaj rad izravno rješava (Tablica 1).

3 Teorijski okvir

Teorija snopova olakšava distribuiranu fuziju signala hvatanjem multimodalnih tokova podataka izravno na uređaju. Perzistentni snopovni Laplaceovi operatori kodiraju i geometrijske i negeometrijske informacije (Wei et al., 2025.), hvatajući višeskalnu fleksibilnost u biomolekularnim sustavima (Hayes et al., 2025.). Ovo svojstvo proširujemo na nestacionarne kardiovaskularne signale.

- Lokalna mišljenja i mape restrikcije: Teorija celularnih snopova oprema senzorsku mrežu geometrijskom strukturom. Sirovi signal koji uhvati senzorski čvor djeluje kao njegovo "privatno mišljenje".
- Operator kogranice: Snopovni Laplaceov operator eksplicitno mjeri agregirano "neslaganje mišljenja" na svakom senzorskom čvoru. Formuliran je korištenjem blok-matričnih operatora kogranice δ_h .

$$L_h = \delta_h^T \delta_h + \delta_{h-1} \delta_{h-1}^T \quad (1)$$

Normalna fiziološka stanja modelirana su tako da borave unutar Harmonijskog prostora ($H^0(G,F) = \ker(L_0)$), predstavljajući globalni konsenzus. Obrnuto, iznenadne anomalije (aritmije ili artefakti kretanja) projiciraju se izvan ove jezgre kao topološka neslaganja. Budući da je ovo izračunavanje vođeno mapama restrikcije koje povezuju susjedne čvorove, operacije se pretvaraju u rijetke rutine linearne algebre koje se udobno smještaju unutar ograničenog SRAM-a na čipu.

4 Zajednički dizajn sklopovlja i programske opreme

Kako bismo izbjegli memorijsko preopterećenje koje uzrokuju Python biblioteke, implementiramo „bare-metal” C/C++ arhitekturu prilagođenu Cortex-M procesorima. Predložena arhitektura prikazana je na Slici 2.

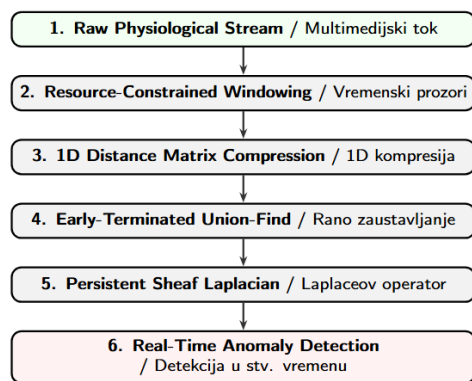


Figure 2. The proposed memory-optimized pipeline executing entirely on-chip.

0	.	.
1	0	.
2	3	0

Discarded (Upper + Diagonal)

Figure 3. Symmetric matrix compression: We index only the lower triangular portion (blue) for a 50% SRAM reduction.

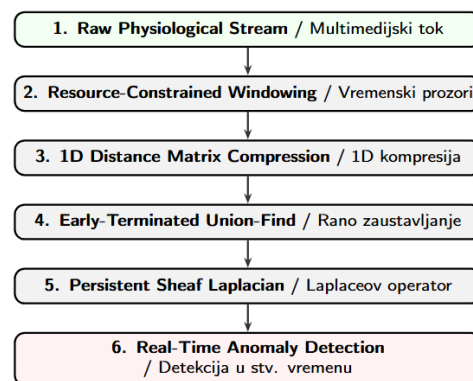
4.1 1D Distance Matrix Compression (Fig. 3)

The allocation for an $N \times N$ distance matrix demands substantial contiguous memory. Since the distance matrix of a real-valued physiological signal is symmetric with a zero diagonal, we flatten the 2D structure into a contiguous 1D array requiring strictly $N \cdot (N-1)/2$ floating-point elements. For $N=64$, the required combinations drop from 4,096 to 2,016, saving precisely 50% of required SRAM.

4.2 Early-Terminated Union-Find Algorithm

To identify the persistence of 0-dimensional topological features (β_0), we deploy a Kruskal-style Union-Find algorithm optimized with path compression. The algorithm executes an early stoppage the moment $\beta_0 = 1$. This threshold effectively caps the underlying simplicial complex and prevents dynamic memory overflow. Stopping at the sheaf global section threshold is mathematically viable for our specific use case because we assume a simplified, constant cellular sheaf, where global graph connectivity directly aligns with the harmonic consensus.

4.3 C++ Architectural Implementation



Slika 2. Predloženi cjevovod optimiziran za memoriju koji se izvršava na čipu.

0	.	.
1	0	.
2	3	0

Discarded (Upper + Diagonal)

Slika 3. Kompresija simetrične matrice: Indeksiramo samo donji trokutasti dio (plavo) za 50%-tno smanjenje SRAM-a.

4.11D kompresija matrice udaljenosti (Slika 3)

Alokacija za matricu udaljenosti $N \times N$ zahtijeva znatnu kontinuiranu memoriju. Budući da je matrica udaljenosti fiziološkog signala realnih vrijednosti simetrična s nultom dijagonalom, izravnavamo 2D strukturu u kontinuirani 1D niz koji zahtijeva strogo $N \cdot (N-1)/2$ elemenata s pomičnim zarezom. Za $N=64$, potrebne kombinacije padaju s 4096 na 2016, štedeći točno 50% potrebnog SRAM-a.

4.2 Rano prekinuti Union-Find algoritam

Za identifikaciju perzistencije 0-dimenzionalnih topoloških značajki (β_0), primjenjujemo Kruskalov Union-Find algoritam optimiziran kompresijom puta. Algoritam izvršava rano zaustavljanje u trenutku kada je $\beta_0 = 1$. Ovaj prag učinkovito ograničava temeljni simplicijalni kompleks i sprječava dinamičko prelijevanje memorije. Zaustavljanje na pragu globalnog presjeka snopa matematički je održivo za naš specifični slučaj upotrebe jer pretpostavljamo pojednostavljeni, konstantni celularni snop, gdje se globalna povezanost grafa izravno podudara s harmonijskim konsenzusom.

To guarantee adherence to stringent SRAM limits, the framework avoids dynamic memory allocation during the continuous monitoring loop. To comply with modern embedded engineering standards, the bare-metal C++ implementation (Listing 1) utilizes memory-safe `std::span` for buffer views and `constexpr` for compile-time evaluation. The symmetric 2D distance matrix is mapped to a contiguous 1D array, and the Union-Find algorithm forcefully breaks the moment $\beta_0 = 1$, locking the memory state.

```

1 #include <span>
2 #include <algorithm>
3
4 // 1. 1D Compressed Symmetric Matrix Indexing (Compile-time evaluated)
5 constexpr uint8_t get1DIndex(uint8_t i, uint8_t j) noexcept {
6     if (i < j) std::swap(i, j); // Ensure lower-triangular
7     return (i * (i - 1)) / 2 + j;
8 }
9
10 // 2. Early-Terminated Union-Find Execution utilizing memory-safe views
11 void computePersistentSheaf(uint8_t N, std::span<const Edge> sorted_edges) {
12     uint8_t betti_0 = N; // Initialize components
13
14     for (const auto& edge : sorted_edges) {
15         uint8_t root_u = findRoot(edge.u);
16         uint8_t root_v = findRoot(edge.v);
17
18         if (root_u != root_v) {
19             parent[root_u] = root_v; // Execute Union
20             betti_0--; // Decrement components
21
22             // Sheaf Global Section Threshold Reached
23             if (betti_0 == 1) {
24                 capSimplicialComplex();
25                 break; // HALT execution: Prevents memory overflow
26             }
27         }
28     }
29 }

```

Listing 1. Modern C++ logic for 1D matrix compression and early-terminated topological reduction utilizing `std::span` and `constexpr`.

5. Results and Evaluation

5.1 Experimental Setup and Real-World Datasets

To validate the real-time feasibility of the proposed framework, physical hardware profiling was simulated using two distinct datasets: The MIT-BIH Arrhythmia Database provided a clinical baseline, while the PPG-DaLiA dataset introduced severe ambulatory motion artifacts (MAs) spanning 0.1 to 10 Hz (Reiss et al., 2019). Execution latencies and SRAM footprints were measured across dynamic analytical windows ($N = 32, 64, 128$).

5.2 SRAM Optimization and The 32KB Threshold

By implementing a 1D symmetric distance matrix compression, our algorithm precisely bounds the required floating-point elements. As demonstrated in Fig. 4, the memory footprint scales predictably: 1.94 KB for $N=32$, and 7.88 KB for $N=64$. Critically, even at the extended window size of $N=128$, the peak memory consumption is strictly capped at 31.75 KB. It is important to acknowledge that this reported 31.75 KB primarily represents the compressed distance matrix. While this proves that the core topological pipeline avoids SRAM overflow within the constrained 32 KB cache limits of ARM Cortex-M

4.3 Arhitektonska implementacija u jeziku C++

Kako bi se zajamčilo poštivanje strogih ograničenja SRAM-a, okvir izbjegava dinamičku alokaciju memorije tijekom petlje kontinuiranog praćenja. U skladu s modernim inženjerskim standardima za ugrađene sustave, “bare-metal” C++ implementacija (Ispis 1) koristi memorijski siguran `std::span` za preglede međuspremnik i `constexpr` za evaluaciju tijekom prevodenja. Simetrična 2D matrica udaljenosti preslikava se u kontinuirani 1D niz, a Union-Find algoritam se prisilno prekida u trenutku kada je $\beta_0 = 1$, zaključavajući stanje memorije.

```

1 #include <span>
2 #include <algorithm>
3
4 // 1. 1D Compressed Symmetric Matrix Indexing (Compile-time evaluated)
5 constexpr uint8_t get1DIndex(uint8_t i, uint8_t j) noexcept {
6     if (i < j) std::swap(i, j); // Ensure lower-triangular
7     return (i * (i - 1)) / 2 + j;
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10 // 2. Early-Terminated Union-Find Execution utilizing memory-safe views
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21
22             // Sheaf Global Section Threshold Reached
23             if (betti_0 == 1) {
24                 capSimplicialComplex();
25                 break; // HALT execution: Prevents memory overflow
26             }
27         }
28     }
29 }

```

Ispis 1. Moderna C++ logika za kompresiju 1D matrice i rano prekinutu topološku redukciju korištenjem `std::span` i `constexpr`.

5. Rezultati i evaluacija

5.1 Eksperimentalno postavljanje i stvarni skupovi podataka

Kako bi se potvrdila izvedivost predloženog okvira u stvarnom vremenu, profiliranje fizičkog sklopovlja simulirano je pomoću dva različita skupa podataka: baza podataka MIT-BIH Arrhythmia pružila je kliničku osnovu, dok je skup podataka PPG-DaLiA uveo teške artefakte kretanja (MA) u rasponu od 0,1 do 10 Hz (Reiss et al., 2019). Latencije izvršavanja i SRAM otisci mjereni su kroz dinamičke analitičke prozore ($N = 32, 64, 128$).

5.2 Optimizacija SRAM-a i prag od 32 KB

Implementacijom 1D kompresije simetrične matrice udaljenosti, naš algoritam precizno ograničava potrebne elemente s pomičnim zarezom. Kao što je prikazano na Slici 4, memorijski otisak skalira se predvidljivo: 1,94 KB za $N=32$ i 7,88 KB za $N=64$. Kritično, čak i pri proširenoj veličini prozora od $N=128$, vršna potrošnja memorije strogo je

microcontrollers, future work will comprehensively profile the total memory state of the application, including signal buffers, sorted edge lists, and sheaf-related matrices.

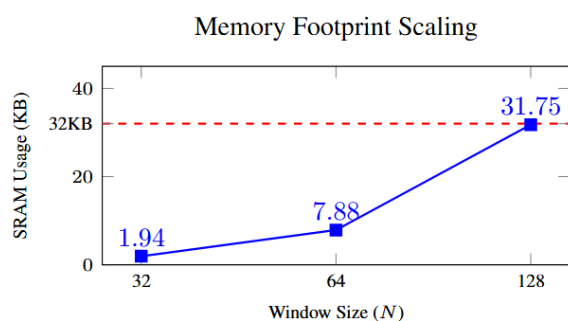
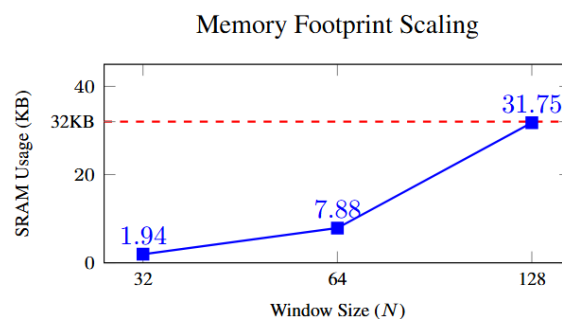


Figure 4. Quadratic scaling of the 1D compressed matrix under the 32 KB threshold

ograničena na 31,75 KB. Važno je napomenuti da ova prijavljena vrijednost od 31,75 KB prvenstveno predstavlja komprimiranu matricu udaljenosti. Iako ovo dokazuje da glavni topološki cjevovod izbjegava prelijevanje SRAM memorije unutar strogih ograničenja predmemorije (cache) od 32 KB kod ARM Cortex-M mikrokontrolera, budući rad će iscrpno profilirati cjelokupno stanje memorije aplikacije, uključujući međuspremnik signala, sortirane liste rubova i matrice povezane sa snopovima.



Slika 4. Kvadratno skaliranje komprimirane 1D matrice ispod praga od 32 KB.

5.3 Execution Latency and Cache Anomalies

The early-terminated Union-Find algorithm yielded exceptional computational latency. As visualized in Fig. 5, execution required only 2.026 ms for MIT-BIH data ($N=64$) and an average of 2.290 ms for the highly noisy PPG-DaLiA dataset. Notably, the profiling revealed a distinct architectural anomaly: the $N=32$ window registered a higher execution time (7.889 ms) than the $N=64$ window (2.026 ms). This inversion is attributed to cold-start execution penalties and processor cache-misses during initial array allocations on the bare-metal architecture. Once the cache pipeline is primed, the framework processes 128-sample multi-channel streams in just 5.06 ms. These results suggest feasibility, while full hardware validation and energy measurements remain future work.

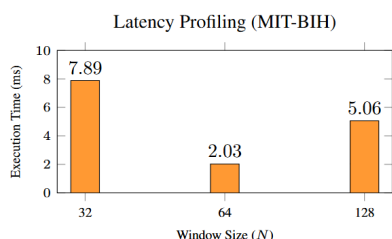
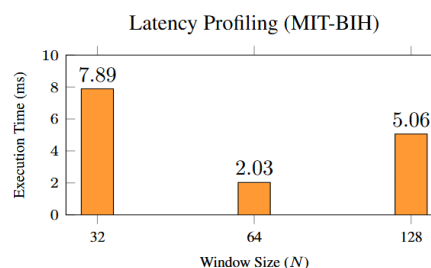


Figure 5. Execution latency across dynamic windows with cold-start anomaly at $N = 32$.

5.3 Latencija izvršavanja i anomalije predmemorije

Rano prekinuti Union-Find algoritam pružio je iznimnu računalnu latenciju. Kao što je prikazano na Slici 5, izvršavanje je zahtijevalo samo 2,026 ms za podatke MIT-BIH ($N=64$) i prosječno 2,290 ms za iznimno šumom opterećen skup podataka PPG-DaLiA. Primjetno, profiliranje je otkrilo jasnu arhitektonsku anomaliju: prozor s $N=32$ zabilježio je dulje vrijeme izvršavanja (7,889 ms) od prozora s $N=64$ (2,026 ms). Ova inverzija pripisuje se penalizacijama izvršavanja zbog "hladnog starta" (cold-start) i promašajima predmemorije (cache-misses) procesora tijekom početnih alokacija nizova na bare-metal arhitekturi. Jednom kada je cjevovod predmemorije pripremljen, okvir obrađuje višekanalne tokove od 128 uzoraka u samo 5,06 ms. Ovi rezultati ukazuju na izvedivost, dok potpuna validacija sklopovlja i mjerenja energije ostaju ciljevi za budući rad.



Slika 5. Latencija izvršavanja kroz dinamičke prozore s anomalijom hladnog starta pri $N = 32$.

5.4 Heterophilic Artifact Isolation in PPG

A deeper clinical analysis of the PPG-DaLiA outputs reveals why the persistent sheaf framework is mathematically necessary. In standard Euclidean networks or traditional Short-Time Fourier Transforms (STFT), high-frequency motion artifacts overlap with cardiac frequencies. This results in oversmoothing, where the network erroneously averages the signal and locks onto the dominant heterophilic noise. Our early-terminated Hodge filtration effectively acts as a geometry-aware high-pass filter, isolating these rapid motion anomalies into localized gradient flows.

While traditional deep learning approaches often rely on heavy feature extraction to report classification metrics like F1-scores, our goal is to validate the mathematical viability of topological features themselves under strict hardware limits. Under multiple linear regression, the isolated harmonic energy fraction (H%) of the cardiovascular wave retains a statistically significant correlation with the ground-truth heart rate ($p = 0.015$). This robust statistical significance provides that the early-terminated sheaf global section successfully captures the true physiological state, providing a rigorous foundation for anomaly detection without the computational overhead of a secondary neural network.

5.5 Comparative Baseline Analysis

To better contextualize the efficiency of the Persistent Sheaf Laplacian, Table 2 compares our approach against standard lightweight deep learning and signal processing techniques under strict microcontroller limits. Unlike Convolutional Neural Networks (CNNs) which demand heavy SRAM for feature maps, or Short-Time Fourier Transforms (STFT) which lack geometric awareness, our pipeline achieves ultra-low memory requirements while preserving heterophilic anomaly isolation.

Table 2. Comparative Baseline: Edge Intelligence Methods for Biomedical Signals

Methodology	SRAM Footprint	Execution Latency	Heterophily Handling
Lightweight CNN (Lin et al., 2020)	High (~217 KB)	> 100 ms	Poor (Oversmoothing)

5.4 Izolacija heterofilnih artefakata u PPG-u

Dublja klinička analiza izlaza PPG-DaLiA otkriva zašto je okvir perzistentnih snopova matematički nužan. U standardnim euklidskim mrežama ili tradicionalnim kratkovremenim Fourierovim transformacijama (STFT), visokofrekventni artefakti kretanja preklapaju se s kardiološkim frekvencijama. To rezultira prekomjernim izgladivanjem, pri čemu mreža pogrešno usrednjava signal i fiksira se na dominantni heterofilni šum. Naša rano prekinuta Hodgeova filtracija učinkovito djeluje kao visokopropusni filter svjestan geometrije, izolirajući ove brze anomalije kretanja u lokalizirane gradijentne tokove.

Dok se tradicionalni pristupi dubokog učenja često oslanjaju na intenzivno izdvajanje značajki kako bi izvijestili o klasifikacijskim metrikama poput F1-mjera, naš je cilj potvrditi matematičku održivost samih topoloških značajki pod strogim ograničenjima sklopovlja. Pod višestrukom linearnom regresijom, izolirani udio harmonijske energije (H%) kardiovaskularnog vala zadržava statistički značajnu korelaciju sa stvarnom frekvencijom srca ($p = 0.015$). Ova robusna statistička značajnost potvrđuje da rano prekinuti globalni presjek snopa uspješno zahvaća stvarno fiziološko stanje, pružajući rigorozan temelj za detekciju anomalija bez dodatnog računalnog opterećenja sekundarnom neuronskom mrežom.

5.5 Usporedna bazna analiza

Kako bi se bolje kontekstualizirala učinkovitost perzistentnog snopovnog Laplaceovog operatora, Tablica 2 uspoređuje naš pristup sa standardnim laganim tehnikama dubokog učenja i obrade signala pod strogim ograničenjima mikrokontrolera. Za razliku od konvolucijskih neuronskih mreža (CNN) koje zahtijevaju velik SRAM za mape značajki, ili kratkovremenih Fourierovih transformacija (STFT) kojima nedostaje geometrijska svjesnost, naš cjevovod postiže iznimno niske memorijske zahtjeve uz istovremeno očuvanje izolacije heterofilnih anomalija.

Tablica 2: Usporedna osnovica: Metode rubne inteligencije za biomedicinske signale

Metodologija	Otisak SRAM-a	Latencija izvršavanja	Upravljanje heterofilijom
Lagani CNN (Lin et al., 2020)	Visoko (~217 KB)	> 100 ms	Slabo (Prekomjerno izgladivanje)

ICA Adaptive Filtering (Martinek et al., 2018)	Medium (> 50 KB)	Iterative	Moderate
Persistent Sheaf (Ours)	Ultra-Low (31.75 KB)	< 8 ms	Excellent (Isolated)

Note: The CNN baseline reflects MobileNetV2 performance via CMSIS-NN (Lin et al., 2020). Traditional ICA reflects standard matrix-heavy filtering (Martinek et al., 2018). ‘Persistent Sheaf’ metrics are derived from our experimental profiling.

5.6 Interactive Edge Hardware Simulator

To facilitate reproducibility and allow the research community to visually explore the dynamic scaling limits of the proposed hardware-software co-design, an interactive web-based simulator was developed (Fig. 6). The dashboard mathematically models the exact static RAM footprint scaling of the 1D compressed distance matrix against the 32 KB physical threshold of Cortex-M architectures. Furthermore, it allows users to dynamically inject heterophilic motion artifacts (mimicking the PPG-DaLiA dataset) into stable rhythmic signals to observe the real-time early-terminated Hodge filtration catching the topological void. The interactive dashboard and associated simulation source code have been temporarily hosted at an anonymized repository: <https://anonymous.4open.science/w/Memory-Optimized-Persistent-Sheaf-LaplaciansJune-D690/>

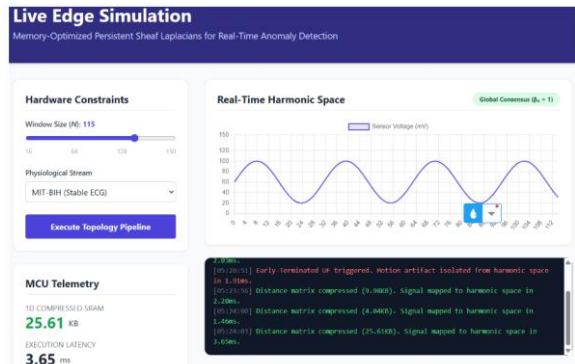


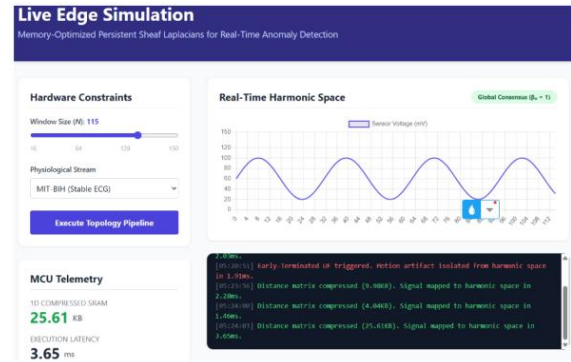
Figure 6. The Interactive Edge CPS Simulator. The dashboard visualizes the mathematical thresholds where unoptimized simplicial complexes trigger 32 KB SRAM memory overflows.

ICA Adaptivno filtriranje (Martinek et al., 2018)	Srednje (> 50 KB)	Iterativno	Umjereno
Perzistentni snop (Naš)	Izuzetno nisko (31.75 KB)	< 8 ms	Izvršno (Izolirano)

Napomena: CNN osnovica odražava performanse modela MobileNetV2 putem CMSIS-NN knjižnice (Lin et al., 2020). Tradicionalna ICA odražava standardno filtriranje s teškim matričnim operacijama (Martinek et al., 2018). Metrike za „Perzistentni snop” izvedene su iz našeg eksperimentalnog profiliranja.

5.6 Interaktivni simulator rubnog sklopovlja

Kako bi se olakšala ponovljivost i omogućilo istraživačkoj zajednici da vizualno istraži dinamičke granice skaliranja predloženog zajedničkog dizajna sklopovlja i programske opreme, razvijen je interaktivni simulator temeljen na webu (Slika 6). Nadzorna ploča matematički modelira točno skaliranje otiska statičkog RAM-a 1D komprimirane matrice udaljenosti u odnosu na fizički prag od 32 KB Cortex-M arhitektura. Nadalje, omogućuje korisnicima da u stabilne ritmičke signale dinamički unesu heterofilne artefakte kretanja (koji oponašaju skup podataka PPG-DaLiA) kako bi promatrali kako rano prekinuta Hodgeova filtracija u stvarnom vremenu hvata topološku prazninu. Interaktivna nadzorna ploča i pripadajući izvorni kod simulacije privremeno su smješteni u anonimiziranom repozitoriju: <https://anonymous.4open.science/w/Memory-Optimized-Persistent-Sheaf-LaplaciansJune-D690/>



Slika 6. Interaktivni simulator rubnog CPS-a. Nadzorna ploča vizualizira matematičke pragove na kojima neoptimizirani simplicijalni kompleksi uzrokuju prelijevanje SRAM memorije od 32 KB.

6 Limitations and Hardware Trade-offs

While this exploratory work resolves the immediate memory bottleneck for pairwise-distance calculations, the framework has inherent limitations. First, the boundary condition of the early-terminated Union-Find algorithm acts as a strict safety mechanism against combinatorial explosion. However, stopping at $\beta_0=1$ discards higher-dimensional topological features (such as 1D cycles or 2D voids). Future work must mathematically quantify this lost analytical resolution using a localized Wasserstein distance metric to rigorously evaluate the trade-off between memory safety and topological fidelity.

Finally, the current evaluation relies on simulated ARM Cortex-M environments. True physical deployment will introduce variable power dynamics and memory bus contentions that simulation cannot perfectly replicate. Consequently, future physical deployments will target specific physical microcontroller boards - such as the STM32L4 and Nordic nRF52840 families - to validate actual power consumption and hardware-in-the-loop performance.

7 Future Directions and Strategic Applications

The successful hardware-software co-design of edge-based sheaf topologies opens several highly strategic vectors for future Cyber-Physical Systems (CPS) research, specifically in advanced biomedical and industrial applications.

7.1 Non-Invasive Fetal ECG and Complex Signal Separation

A critical future application of this framework is the real-time extraction of Non-Invasive Fetal Electrocardiograms (NI-FECG) from abdominal sensors. Current state-of-the-art methods rely heavily on Independent Component Analysis (ICA) and adaptive filtering to separate fetal and maternal signals (Martinek et al., 2018; Jaros et al., 2020). However, these statistical methods often struggle with the overlapping spectral frequencies of maternal and fetal cardiac cycles in highly noisy ambulatory environments. Because the persistent sheaf Laplacian acts as a non-parametric, geometry-aware filter, future iterations will explore using restriction maps to topologically isolate the high-frequency fetal QRS complexes (treated as topological voids) from the dominant maternal cardiac cycle (the harmonic consensus). Achieving this on-edge without the

6 Ograničenja i kompromisi sklopovlja

Iako ovaj istraživački rad rješava trenutačno memorijsko usko grlo za izračune udaljenosti po parovima, okvir ima inherentna ograničenja. Prvo, granični uvjet rano prekinutog Union-Find algoritma djeluje kao strogi sigurnosni mehanizam protiv kombinatorne eksplozije. Međutim, zaustavljanje na $\beta_0=1$ odbacuje višedimenzionalne topološke značajke (kao što su 1D ciklusi ili 2D praznine). Budući rad mora matematički kvantificirati ovu izgubljenu analitičku rezoluciju pomoću lokalizirane metrike Wassersteinove udaljenosti kako bi se rigorozno procijenio kompromis između memorijske sigurnosti i topološke vjernosti.

Naposljetku, trenutačna evaluacija oslanja se na simulirana okruženja ARM Cortex-M. Stvarna fizička implementacija uvest će varijabilnu dinamiku potrošnje energije i nadmetanja na memorijskoj sabirnici koje simulacija ne može savršeno replicirati. Stoga će buduće fizičke implementacije ciljati na specifične fizičke mikrokontrolerske ploče - poput obitelji STM32L4 i Nordic nRF52840 - kako bi se potvrdila stvarna potrošnja energije i performanse u petlji sa sklopovljem (hardware-in-the-loop).

7 Budući smjerovi i strateške primjene

Uspješan zajednički dizajn sklopovlja i programske opreme za rubne topologije snopova otvara nekoliko iznimno strateških vektora za buduća istraživanja kibernetičko-fizičkih sustava (CPS), specifično u naprednim biomedicinskim i industrijskim primjenama.

7.1 Neinvazivni fetalni EKG i razdvajanje složenih signala

Ključna buduća primjena ovog okvira je izdvajanje neinvazivnih fetalnih elektrokardiograma (NI-FECG) u stvarnom vremenu iz abdominalnih senzora. Trenutačne najsuvremenije metode uvelike se oslanjaju na analizu nezavisnih komponenti (ICA) i adaptivno filtriranje kako bi se razdvojili fetalni i majčini signali (Martinek et al., 2018.; Jaros et al., 2020.). Međutim, ove se statističke metode često bore s preklapajućim spektralnim frekvencijama majčinih i fetalnih srčanih ciklusa u iznimno šumom opterećenim ambulantnim okruženjima. Budući da perzistentni snopovni Laplaceov operator djeluje kao neparametarski filtar svjestan geometrije, buduće iteracije istraživat će korištenje mapa restrikcije za

computational overhead of iterative ICA matrices represents a major leap for continuous, low-power maternal-fetal monitoring.

7.2 Smart Mobility and Automotive CPS

Beyond wearable medical monitors, this hardware-optimized framework has immediate applications in Smart Mobility. We plan to adapt the algorithm for automotive cyber-physical systems, specifically for continuous driver fatigue and anomaly monitoring using steering-wheel-embedded PPG and ECG sensors. Processing these multi-channel signals locally on automotive microcontrollers ensures zero-latency safety interventions independent of 5G cellular network stability.

7.3 Industrial IoT and the Edge-to-Cloud Continuum

Because the sheaf Laplacian natively compresses distributed sensor signals into lightweight topological summaries (Betti numbers and harmonic fractions), it is an ideal candidate for large-scale Industrial IoT (IIoT) and predictive maintenance. Future research will explore deploying these algorithms on ultra-low-power industrial sensors to detect acoustic and vibrational micro-anomalies in heavy machinery. Furthermore, instead of transmitting raw telemetry, edge nodes will push these minimal topological summaries to centralized supercomputing infrastructures for integration into global Digital Twins, optimizing the edge-to-cloud continuum.

8 Conclusion

This paper introduces a memory-optimized Persistent Sheaf Laplacian algorithm designed specifically for resource-scarce embedded platforms. By compressing spatial distance computations into a 1D matrix and deploying an early-terminated Union-Find execution, we restrict dynamic SRAM consumption to under 32 KB while processing multi-channel signals in under 8 ms. The preliminary results suggest that this hardware-software co-design makes selected topological feature extraction operations computationally feasible on memory-constrained embedded platforms. This lays the groundwork for future physical hardware validation, total memory profiling, energy measurements, and task-level anomaly detection in cyber-physical systems.

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topološku izolaciju visokofrekventnih fetalnih QRS kompleksa (tretiranih kao topološke praznine) od dominantnog majčinog srčanog ciklusa (harmonijskog konsenzusa). Postizanje ovoga na samom rubu bez računalnog opterećenja iterativnih ICA matrica predstavlja golem iskorak za kontinuirano praćenje majke i fetusa uz nisku potrošnju energije.

7.2 Pametna mobilnost i automobilske CPS

Osim nosivih medicinskih monitora, ovaj sklopovski optimizirani okvir ima izravne primjene u pametnoj mobilnosti. Planiramo prilagoditi algoritam za automobilske kibernetičko-fizičke sustave, specifično za kontinuirano praćenje umora vozača i anomalija pomoću PPG i EKG senzora ugrađenih u upravljač. Lokalna obrada ovih višekanalnih signala na automobilske mikrokontrolere osigurava sigurnosne intervencije s nultom latencijom, neovisno o stabilnosti 5G mobilne mreže.

7.3 Industrijski IoT i kontinuum od ruba do oblaka

Budući da snopovni Laplaceov operator izvorno komprimira distribuirane senzorske signale u lagane topološke sažetke (Betti-jeve brojeve i harmonijske udjele), on je idealan kandidat za industrijski IoT (IIoT) velikih razmjera i prediktivno održavanje. Buduća istraživanja istraživat će primjenu ovih algoritama na industrijskim senzorima iznimno niske potrošnje energije radi otkrivanja akustičkih i vibracijskih mikroanomalija u teškim strojevima. Nadalje, umjesto prijenosa sirove telemetrije, rubni će čvorovi slati ove minimalne topološke sažetke u centralizirane superračunalne infrastrukture radi integracije u globalne digitalne blizance, čime se optimizira kontinuum od ruba do oblaka.

8 Zaključak

Ovaj rad predstavlja memorijski optimiziran algoritam perzistentnog snopovnog Laplaceovog operatora, dizajniran specifično za ugrađene platforme s ograničenim resursima. Komprimiranjem izračuna prostornih udaljenosti u 1D matricu i primjenom rano prekinutog Union-Find izvršavanja, ograničavamo dinamičku potrošnju SRAM-a na manje od 32 KB, uz obradu višekanalnih signala za manje od 8 ms. Preliminarni rezultati ukazuju na to da ovaj zajednički dizajn sklopovlja i programske opreme čini odabrane operacije izdvajanja topoloških značajki računalno izvedivima na memorijski ograničenim ugrađenim platformama. Time se postavljaju temelji za buduću validaciju na fizičkom sklopovlju, cjelovito profiliranje memorije, mjerenja

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