Tangible User Interfaces for Enhancement of Young Children's Mathematical Problem Solving and Reasoning: A Preliminary Review of Relevant Literature

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Abstract. The aim of this paper is to explore how Tangible User Interfaces (TUIs) have been used in enhancement and learning activities that support mathematical learning of young children. The research is based on the existing scientific literature, models and frameworks. Conducted research impliesthat TUIs can support mathematical problem solving and mathematical reasoning, but further theoretical and empirical research is needed in order to identify specific properties of TUIs that benefit learning.

Keywords. Tangible User Interface, Education, Mathematics, Young children

1 Introduction

In the 21 century it is necessary to provide a firm mathematical knowledge and competence to young children so that they may be successful in STEM (Science, Technology, Engineering and Mathematics) disciplines. The growing impact of technologies on our everyday life has reshaped children's models, methods, forms and frames of learning. For this reason, many EU countries have recognized the importance of technologies in education and have renewed the core curricula for education, starting from primary education. For instance, Finland has renewed the national core curricula to update the comprehensive school system to the 2020 requirements to make Finland the number one country of inspiring learning and education (Kimmo, 2017). The reform focuses on three things:

- new pedagogy,
- new learning environments and
- digital learning.

Croatia has also recognized the importance of technologies and their use in early education. *The National Curricula for Early Childhood and Preschool Education* provides a framework to enhance all developmental domains in accordance with each **Andrina Granić**

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child's abilities (Slunjski *et al*., 2014). The emphasis is on development of competences for lifelong learning which include mathematical competence and digital competence. In this paper we will consider young children to be children aging from 0 to 11 years in accordance to Piaget's stages of cognitive development (Piaget, 1964.). To explore possible learning benefits of learning with and about technologies for young children, we must consider cognitive theories and pedagogical practices which imply what are the most suitable forms of interaction.

Interaction with technology should enhance development of abstract mathematical concepts. One such form of interaction can be, for example, manipulation of objects. We must also examine practical application of above mentioned theories and practices in technology. Most common application are so called Tangible User Interfaces (TUIs), which will be discussed with regards to their use in learning.

In this paper we will review relevant scientific publications that consider the use of TUIs for enhancement of young children's mathematical problem solving and mathematical reasoning. TUIs might prove to be the most beneficial for learning in this domain, because trough linking of physical materials with digital information, children can explicitly see the relationship between concrete and abstract concepts.

2 Cognitive theories and pedagogical practices

If we are to design and create technologies for children, we must consider researches and theories on child development. Jean Piaget's theory of cognitive development is one of the major contributions of the 20th century to developmental psychology and education. Piaget (1964) distinguishes four stages of cognitive development:

- 1. sensory motor stage (from 0 to 2 years),
- 2. preoperational stage (from 2 to 7 years),
- 3. concrete operations stage (from 7 to 11 years),
- 4. formal operations stage (from 11 to 15 years).

Piaget thought that learning occurs through a process of children's adaptation to their environment. He considered adaptation as an active process which occurs by children's interactions with the world around them; with interactions children gain experience and construct knowledge (Hourcade, 2015).

The idea and the approach claiming that is important to learn through experience rather than being told about it, was laid down by educator Maria Montessori (1912). In her work she observed children in their activities and it helped her design special didactic materials. The materials are learning objects, such as wooden blocks, which allow purposeful learning activities and multi-sensorial interactions. The goal of those learning objects is to maximize children's learning potential (Montessori, 1912). Nowadays, these objects are called *Manipulatives* (Antle, 2013) physical objects specifically designed to foster learning.

The physical manipulation of objects requires not only physical but also mental activities; in addition, it plays a crucial role in the development of thinking skills in general. Manipulation lightness the cognitive load by simplifying abstract concepts and makes them more accessible to young children (Antle and Wise, 2013).

A modern approach to Montessori didactic materials is provided by (Zuckerman *et al*., 2005) offering a new classification of Manipulatives - *Montessori-inspired Manipulatives* (MiMs). They argue that MiMs foster modelling of more abstract structures. The research suggested that *Digital MiMs,* shown in Figure 1, are engaging learning environments despite them being abstract, and they give children an opportunity to interact with dynamic behaviour at the symbolic level rather than the example level. They showed that Digital MiMs promote group interaction and discussion.

Figure 1. Illustration of MiMs

Consequently, we may assume that concrete physical manipulation of objects might support children's effective or natural learning thus allowing them to focus on the core of the problem that needs to be solved (Marshall, 2007).

3 Tangible User Interfaces and learning

Modern technologies such as mobile phones or computers are "paths" which lead us into the digital world allowing us to seek and give information. However, they are commonly designed for adult users and thus, not always suitable for young children. In the light of the previously mentioned possible benefits of touching and manipulating objects for learning enhancement, we might consider technologies that provide same models of interaction.

Tangible User Interfaces (TUIs) are interfaces that give physical form to digital information. The term TUIs was proposed by Ishi and Ullmer in 1997, who took their inspiration from the ancient counting tool, abacus (Ullmer, 2002). With TUIs we have a seamless integration of corresponding digital representation and digital control (Antle, 2013). TUIs provide the opportunity to reshape educational technologies in accordance to cognitivist theories of learning and can be used to support different types of learning (Markova et at., 2012). Marshall (2007) gave six perspectives on how to regard the use of TUIs with respect to learning: possible learning benefits, typical learning domains, exploratory and expressive activity, integration of representations, concreteness and sensory directness as well as effects of physicality. However, there is still little empirical work that provides evidence to claim that TUIs enhance learning, and moreover, there is a lack of theoretical framework that outlines how different features of TUIs affect learning outcomes, especially when designing guidelines (Antle and Wise, 2013).

A review of TUIs for learning was done by Markova. She proposed a classification framework that attended the important aspects of TUIs such as type of interaction or type of object manipulation in addition to their use in learning, for example explicit learning of facts tended to be mostly achieved using TUIs (Markova, 2012). However, she did not give a particular framework that considered children's use of TUIs for learning.

The model that focusses especially on TUIs for children is the *Child Tangible Interaction* (CTI) *framework* by Antle from 2007. The CTI framework is a conceptual framework for the design of tangibles and interactive spaces which support schemata level knowledge acquisition in children (Antle, 2007, p. 2). This framework focuses primarily on children above the age of four and under the age of twelve and is presented in five themes: Space for Action, Perceptual Mappings, Behavioural Mappings, Semantic Mappings and Space for Friends. These five themes of the CTI framework define vertical research areas for tangible and spatial interaction and children.

With regards to Marshall perspective, Antle and Wise presented the *Tangible Learning Design Framework* in the 2013. This framework is compiled from a taxonomy of five elements that need to be considered when relating TUIs features, interactions and learning. Specifically, those elements are physical objects, digital objects, actions on objects, informational relations and learning activities. For each element, guidelines for design are also provided. Altogether, the taxonomy and the guidelines constitute the *Tangible Learning Design Framework* (Antle and Wise, 2013).

Studies are needed that will explore if the Tangible User Interfaces are truly beneficial for children's learning. It needs to be explored how different interaction styles facilitate the development of children's problem solving skills and, if done in groups, how communication skills improve while they solve problems (Antle, 2013). It is also acknowledged that there is a clear requirement for researches to focus on the long-term effects of learning with TUIs in the classroom settings (Markova, 2012).

4 Research

4.1. Motivation

Mathematics enables problem solving in various areas of science and real life. Traditional maths teaching focuses more on giving procedural knowledge and less on applications of these knowledge in real world (Volk *et al*.,2017). In the last decade the use of technology to support learning has increased. The National Council of Teachers of Mathematics of the USA emphasized the importance of technology in teaching and learning mathematic, since it influences the way maths is taught and enhances learning. (Moller, 2015).

Tangibility and tangible interactions and real life observations seem to play an important role in mathematics, but we still lack formal evidence that tangible enhances learning (Marichal, 2017).

In theory TUIs could be beneficial because children are not explicitly taught about the link between abstract

or symbolic content and its concrete physical manifestation (Moller, 2015). We need to understand relations between physical actions and cognitive processes, the link between physical and digital elements through actions, the system feedback and the impact of these elements on the problem solving processes (Marichal, 2017). To better understand these relations and give future guidelines for design of TUIs that might enhance mathematical learning and problem solving, we have conducted a review of prior relevant literature since this is a crucial feature of any scientific study.

4.2. Method

In order to identify relevant scientific publications, a focused structured approach following the suggestions of Webster and Watson (2002) was adopted:

- 1. search of the set of keywords,
- 2. refinement of publications by title,
- 3. quick scan of selected publications by abstracts and
- 4. detailed analyses of full texts.

The search of the literature was done through the search of the Web of Science database and the ACM Digital Library. It was carried out during a month period from April until May 2018. Set of keywords consisted of words *child/children, tangible/tangibles, touch, interface/interfaces, interaction, and math/mathematics* that were joined with AND and OR operators giving the final search phrase:

"child*"AND[("tangibl*"OR"touch")AND ("interaction"OR"interface*")AND("math*"]

The truncation method was used to cover all variations of keywords; for example, child* was used to search for literature that included the word child or children, while math* was used to search for literature that included the word math or mathematics. The time span of the search was not limited, so we took all time span, and as the first result 80 publications were selected. Figure 2 offers an illustration of the search for the Web of Science database.

Figure 2. Illustration of the Web of Science search

Publications	Research method (s)	Main findings and conclusions
Saavedra A. Shoemaker A. (2017).	With the intention of promoting math's big	This first version of DiMBI promotes the successful
DiMBI: An Interface to Connect People	ideas in formal and informal settings, we	accomplishment of the learning goals. In the short
to Math's Big Ideas of Patterns and	created DiMBI (Discovering Math's	term, we plan to find other physical mechanisms
Relations. In Proceedings of the 2017	multimedia platform developed in Processing	similar to those within a Rubik's cube to promote
Conference on Interaction Design and Children	that uses reacTIVision to read users'	our assisted flexibility principle. We believe this
(IDC '17). ACM, New York, NY, USA,	interactions with selected tangible regular	added structure might assist users in developing and
721-724	polygons (a triangle and a square). The	testing meaningful schemata, thereby facilitating
	corners of each polygon are linked with	deeper understanding of the learning objectives.
	specific features that aim to promote transfer	Once we have this mechanism, we plan to conduct
	learning (colours and music). The features	user test sessions in formal and informal learning
	change as the user manipulates the objects	environments during three stages: interviews, think
	through symmetric actions and permutation	aloud, and focal groups.
	of corners.	

Figure 3. Example of an information related to the selected publication

The publications were selected by title and then further analysed through a detailed process of reading abstracts and full texts. To ensure that the results were up to-date, daily e-mail alert was activated about new entries for the saved search. Some of the publications were excluded and the main criteria were the following: if publications were focused on different age groups of children (children older than 11 years), if they were phycology or socially oriented, if they regarded children with special needs, if they did not emphasize the use of TUIs in mathematical education, if they were not in English, if they were mainly technological publications in the sense that their main focus was engineering, or if they were focused on more traditional use of technology such as traditional input devices.

Finally, 16 scientific publications were selected for this review, providing a time span from 2001 up to year 2017. Publications were then organized in a single table, offering insight into relevant information of the publication itself (author(s), title, journal/conference info, publication year), applied research methods along with main findings and conclusions (see Figure 3). Such organization if selected publications enabled the creation of the main research focus centred table.

5 Results and discussion

The 16 selected publications were organised in a main research focus table, presented in Table 1. In the table every publication appears only once, although some of them could be focus on various concepts. We analysed full texts and based on their aspects of use of TUIs that support young children's mathematical problem solving and mathematical reasoning, we were able to distinguish four main research focus areas:

- design and/or implementations of a learning system or application,
- behavioural or cognitive change,
- enhancement of numerical or arithmetical abilities and
- theoretical review.

Most of the publications focused their research on *design and/or implementations of a learning system or application* (10 out of 16) which makes 62.5% out of all selected publications. Among them, five (Marichal *et al*. (2017), Kubicki *et al*. (2016), Barendregt *et al*. (2012), Leong and Horn (2011) and Khandelwal along with Mazalek (2007)) focused on design and implementation, four focused only on the design (Zanchi et al. (2013), Bumbacher *et al*. (2013), Masood and Hoda (2014) plus Saavedra and Shoemaker (2017)), while just one (Scarlatos and Landy (2001)) focused only on the implementation. This finding implies that researchers tend to focus on a design *and* implementation of a learning system or application thus offering insight into the overall development process.

Among the publications that are focused on the design and implementation, the one that is explicitly related to *Tangible Learning Design Framework* (Antle and Wise, 2013) will be introduced in the following. Specifically, Marichal *et al*. (2017) presented CETA (Ceibal Tangible), a mixed-reality system with tangible interaction for 5-6 year old children. This mixed reality environment for mathematical learning is inspired in OSMO, a mixedreality play system for iPads. The authors discussed the design of CETA system in terms of the five element taxonomy proposed in the *Tangible Learning Design Framework*.

CETA is composed of an Android low cost tablet, a mirror, a holder and a set of wooden blocks which play the role of manipulatives. The goal of the game is to learn the concepts of additive composition and the number line representation (*learning activity*). The game narrative is about a robot called Bruno that needs to collect some screws appearing at a certain distance from it. Using the blocks, children must compose the number that matches this distance. Once they put the blocks on the table, the robot will perform an action to pick the screw. *Physical objects* are wooden blocks that become digital manipulatives through markers; each physical block is virtually represented through a virtual block (*digital object*) with the same color and shape on the screen. The most relevant digital object is the main character of the game, robot Bruno. Children control Bruno's actions and movements combining the blocks as illustrated in Figure 4.

As for the *actions on the objects*, children can move the blocks freely, although not all sensible actions for them are sensible or desirable for the system. Regarding *informational relations*, the mappings between physical objects, digital objects and actions, which can be perceptual (physical objects representing digital objects) or behavioral (specific actions on physical objects impacts on digital objects), are considered.

Figure 4. Children playing with CETA.

Going back to the overview of 16 selected papers, with regards to the frequency of publications by year, Figure 5 shows a number of articles that were published in a specific year.

We can notice that the most prominent year was 2017. Five publications were published from 2001 to 2012 and 11 from 2012 to 2017. The number for the latter period is more than 2 times higher than that of the former period indicating an increasing interest on this topic in the last five years.

Figure 5. Frequency of publications

Further analyses of selected publications, enabled us to outline three factors that distinguish selected research: *learning topic*, *environment of the conducted experiment* and *form of the tangible object*.

Bearing in mind the *learning topic*, selected publications considered the following five topics: algebra, arithmetic, number line and/or cardinality, time and orientation as well as geometry. Table 2 shows the cross-analysis results of the research focus and learning topic.

From the achieved results we may conclude that *arithmetic* outnumbers all other learning topics and it is the most frequent one with respect to design and/or implementation of a learning system or application. In contrast, *algebra* and *time and orientation* are learning topics that are least frequent. Reason for such small frequency probably lies in the complexity of algebra. However, algebra is one of the fundamental mathematical branches because it includes everything from elementary equation solving to the study of highest abstracts. Algebra is a unifying thread of almost all of mathematics and there is a need for further research involving this learning topic with respect to all research focuses.

Among the selected publications, 13 of them have conducted experiments and 11 involved young children. However, in two publications (Masood and Hoda (2014) and Bumbacher *et al*. (2013)) authors pointed out that due to the lack of time they were unable to conduct empirical research with children although initially it was planned. The experiments that involved young children had two distinguishing environments: *formal school environment* and *informal environment*. The experiments that were done in *formal school environment* were Kubicki *et al*. (2016), Barendregt *et al*. (2012), Leong and Horn (2011), Scarlatos and Landy (2001), Mock *et al*. (2016), Jong J-T *et al*. (2013), Volk *et al*. (2017). Experiments done in *informal environment* were Marichal *et al*. (2017), Khandelwal and Mazalek (2007), Sedaghatjou and Campbell (2017) and Roberto and Teichrieb (2012). The results imply that experimental work tend to be conducted in the *formal school environments*. We would like to point out that in the aforementioned publications the authors did not explain why a particular environment was selected for the experiment.

Regarding the *form of the tangible object*, three forms by which the children interacted could be differentiated in selected publications: *manipulatives*, *tablet* and *tabletop* (see Figure 6). The form of the tangible object is closely related with the learning topic of the conducted research implying that it shapes the form of the tangible object. Table 3 shows a crossanalyses of frequencies of use of the form of the tangible object with respect to the learning topic.

From the results we may conclude that tablets and tabletops outnumber the use of manipulatives when learning topic is arithmetic. However, manipulatives seem to have a broader range of use, since they are evenly used in all learning topics and are most frequent form of the tangible object. This is consistent with previously mentioned theories of Piaget and Montessori because of physical and spatial affordances of manipulatives. As stated before, children develop cognitively from physical engagement in reasoning with materials in real world settings.

6 Conclusion

Finally, we may draw several implications on the influence of Tangible User Interfaces (TUIs) on young children's mathematical problem solving and reasoning. Research indicate that digital systems and game based learning motivates children, engages them into collaboration as well as enhances successful task completion. (Marichal *et al*. 2017), (Volk *et al*., 2017], (Jong *et al*., 2013). Moreover, the research indicates that children achieve higher learning performances while using TUIs compared to other forms of interaction (Jong *et al*., 2013). One of the advantages of TUIs is that they can provide an external record of previous states and actions and may provide a huge potential for enhancing numerical learning and should thus be explored in future studies (Moeller *et al*., 2015).

However, there is a lack of empirical work which can provide evidence for enhanced learning in mathematics by young children (Chaliampalias, 2016) In a more general note, we conclude that researches and learning specialists need to further their research considering TUIs for learning in order be used in formal school environments (Markova, 2012).

Furthermore, there is a need for a long term exploration of benefits that TUIs may have for enhancement of young children's mathematical problem solving and reasoning. There is also a need for a concrete design framework for the use of TUIs in math education. Such a framework should provide the designers and researchers with design guidelines from cognitive theories and pedagogical practices. These guidelines should point out the appropriate form of tangible object and models of interaction that will facilitate better learning outcomes.

Figure 6. Examples of different forms of a tangible object: **a)** manipulatives, **b)** tablet, **c)** tabletop

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