Modelling the size of underground economy in Croatia: fuzzy logic approach

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Abstract. *The size of the 'underground economy' (UE) is valuable information in the formulation of macroeconomic and fiscal policy. From a scientific point of view analysis of the UE is faced with severe data problems because underground activities are not recorded and anyone engaged in it has an incentive to hide them. This paper estimates the size of the Croatian underground economy by applying novel methodological approach: fuzzy logic. Two major factors affecting the size of the UE, unemployment rate and number of registered business subjects, are used. Results provide insight into movement of Croatia`s UE in the last 10 years.*

Keywords. Underground economy, unemployment, registred businesses, goverements acts, fuzzy logic.

1. Introduction

During a long period of time the Republic of Croatia was faced with negative economic trends which resulted with high unemployment rate and increased rate of the poverty. Such negative trends in the economy led to the increased size of the underground economy. This paper models the movements of underground economy index in Croatia for a period of ten years: from 2003 to 2013 through monitoring of two factors: the level of unemployment rate, and thus the social exclusion of individuals, and changes in the number of registered crafts and companies. The results will be compared with data on the number of operators in the performance of unregistered activities at competent state authority[1].

In the introduction we explain the term "underground economy" and forms of the underground economy, which will be the subject of this study. The term itself has a broad meaning and different authors define it according to their views. The underground economy is described in four forms, defined by Feige [10]:

- Illegal economy - including manufacturing and distribution goods and services forbidden by law, such as drugs, prostitution, smuggling, organized crime, usury, theft,
- Undeclared economy - activities which avoids some fiscal rules in the tax laws, we can talk about the evasion of taxes and contributions,
- Unregistered Economy - includes activities that are not reported to official statistics, we can say that is measured by the income that is not recorded in the national accounts,
- Informal economy-activities which reduce company costs and violate administrative rules which regulate property rights, employment contracts, loan contracts, social security.

At the national or international level we are talking about economic activities of individuals, informal groups or organizations that are not publicly declare (undeclared economy), it is often not recorded (unregistered econom) or not taxed (evasion) [17].

These are the economic activities of organizations or individuals that do not comply with certain institutional procedures, or elude some components of social regulations including activities that are explicitly banned or their execution is related to crime. With respect to the defined forms of the UE this paper will follow the form of the UE "unregistered economy", which includes performing of activities by companies or persons which are not registered by the competent authority.

The Act on the Prohibition and Prevention of performing unregistered [1] was adopted in 2011 and it`s one of the legislative measures of state policy aimed to reduce this form of labor and thus combating the undreground UE [1]. State Inspectorate, as the government institution, is in charge for law enforcement and identifying violations of activities without registering. Other measures which the government is trying to impact on reducing such negative trends are forms of business registration in the relevant institutions and various financial

[1] The competent state authority-national authority for the implementation of inspection supervision-the State Inspectorate, ceased to operate 31.12.2013, their jobs took over Ministry of Finance [6]

incentives. This paper seeks to identify the extent of UE in Croatia, find the factors affecting it and suggest measures to reduce it.
The paper is organized as follows. Next chapter gives an overview of previous research regarding underground economy. Third chapter describes fuzzy logic and data used in the research to estimate size of UE. The fourth chapter discusses empirical analysis and provides the results of estimated UE level in Croatia for 10 years period. Fifth chapter summarizes measures to reduce the impact of the underground economy and sixth chapter concludes the paper.

2 Review of related studies

The UE as negative economic phenomenon occurs in developed and developing countries. But the difference is that in the countries with impoverished economy, as it is currently in Croatia, it has a big impact on the decline in the country's GDP. Reasons for the emergence of the UE can be classified into: economic factors, psychological factors and factors of opportunity.
The economic factors are financial problems of people including the current high rate of unemployment, high taxes, the likelihood of identifying (hope that tax evasion will not be revealed), severity of sanctions (hope that sanctions will not be severe), the expected profit (high profits due to high risk), etc [17].
The psychological factors may include distrust in the country and it`s economic measures, disagreement with the objectives and means of economic policy and attitude towards risk.
As the most important factors of opportunity are work experience and education which is important in finding a job and successfully performing the job beyond the scope of legislation.
In fact, at the times when UE is spred most vulnerable are taxpayers. In other words, because of the increasing size of the UE is very difficult to perform normal registered activity, because the work in UE makes it impossible. The price of goods or services is under the influence of labor in the UE and decreases, meanwhile taxpayers` benefits to the state will remain the same. Here is an example of providing car repair business when a person who does not have a registered business and does not pay taxes, can do a favor for a third party at a lower price than the subject which is registered by the relevant authorities and who is a taxpayer. Furthermore, poverty in the country and the increasing rate of unemployment rate force the individuals to use such services at cheaper subjects who is an unregistered taxpayer. Such economic trends lead to the growing volume of UE. It is therefore important to constantly draw attention of people against this form of deterioration of the economy, that is still insufficiently known area. In addition psychological factors may also play an important role in the development of the UE because in certain societies such as Croatian, people are still reluctant to report illegal performance to the relevant services.

Previous scientific research investigated various elements which can predict the level of UE, such as: unemployment rate [11], the number of registered businness subjects [4], the corruption of government [16] and crime rates [9]. Unemployment rate and number of registered businness subjects are considered to be the major ones and we use it in our research.

Many different methods have been used in the various empirical studies of the underground economy that have been undertaken so far. Trend of using "*Multiple Indicator, Multiple Causes*" (MIMIC) modelling to estimate the size of the underground economy was noticed. However, this approach has been criticized on econometric grounds by Breusch [5]. In order to avoid that and the need for rigid mathematical modeling and distribution assumptions, this study uses fuzzy set theory and fuzzy logic to estimate the size of Croatian`s UE over the period from 2003 to 2013. This gives one of the most valuable contribution of these paper: use of a novel methodological approach in econometrics: fuzzy logic approach. To our knowledge there are only two such contributions: those of Draeseke and Giles from 2002. and Yu, Wang and Chen from 2006. Draeseke and Giles measured the level of UE in New Zealand [8], wheres Yu et al. measured UE in Taiwan [19]. Our paper represents first attempt to apply complex fuzzy logic methodology to measure level of underground economy in European context.

3 Estimation methodology

Empirical analysis in this research uses fuzzy set theory and fuzzy logic models. Fuzzy logic has become an important tool for a number of different applications ranging from the control of engineering systems and artificial intelligence to social sciences. Contrary to traditional approach, fuzzy set theory allows partial or staggering membership of the element to the set. In contrast to the conventional parameter method, fuzzy logic avoids the need for rigid mathematical modeling and the distribution assumption. Fuzzy logic translates natural language descriptions of decision policies into an algorithm using a mathematical model.
Fuzzy logic is based on idea of fuzzy set. Fuzzy set consists of pair: value and membership degree. A degree of membership assume values between 0 and 1 and indicates the degree to which the value of belongs to a set. Fuzzy modelling consists of three phases: fuzzification, inference and defuzzication [20]. The purpose of fuzzification is to map the inputs (rate of unemployment and number of registered subjects) to values from 0 to 1 using a set of input membership functions. These input membership functions represent fuzzy concepts such as "high" or "low".

After the inputs are fuzzified, we know the degree to which each part of the antecedent is satisfied for each rule and we can perfom fuzzy inference. If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule [20]. This number is then applied to the output function. The input to the fuzzy operator is two or more membership values from fuzzified input variables. To obtain the crisp number (UE index) as a result of a process we are performing defuzzification.

3.1. Data description

Data of unemployment rate and number of registered business subject in Croatia for the period of 2003 to 2013 are collected from the Croatian Bureau of Statistics. Summary of the collected data is given in table 1.

Table 1. Unemployment rate and number of registered business subjects in Croatia from 2003. to 2013.

	UEMP	BS
2013	20,3	209 770
2012	18,9	200 264
2011	17,8	195 355
2010	17,4	201 926
2009	14,9	198 478
2008	13,2	204 159
2007	14,8	196 783
2006	16,6	189 231
2005	17,9	178 521
2004	18,0	178 056
2003	19,5	168 425

The data on underground economy index was constructed for annual time series for the period 2003 to 2013 using the fuzzy logic approach. Such a model consists of fuzzification, inference, and defuzzification [20]. In the present study, we follow closely the work of Draeseke and Giles [8].

4 Empirical analysis

We choose factors with a major influence in terms of measuring the UE: unemployment rate and number of registered business subjects. First, we set up the membership functions for the two factors. Both factors are described using linguistic terms: Very Low (VL), Low (L), Normal (N), High (H), and Very High (VH). Each term is associated with membership functions. The boundaries of membership functions are calculated by using the moving average of the past consecutive years (for term Normal) and by adding or subtracting one or two standard deviations (SD) as seen for factor UEMP in table 2.

Table 2. Linguistic terms of UEMP

UEMP: uneployment rate				
Very Low (VL)	Low (L)	Normal(N)	High(H)	Very High(VH)
Mean-2SD	Mean-1SD	Mean	Mean+1SD	Mean+2SD

In this way two sets, each of five numbers corresponding to UEMP and BS are generated for each year. Consider the data from table 1 for year 2010. Value for UEMP is 17,4 which is somewhere between normal and high. How normal and how high it is depends on its location relative to the boundaries. In fuzzy logic the establishment of levels of associations is achieved by using *membership functions*. Membership functions are presented in table 6 and figure 3 in appendix. UEMP in 2010 has following values of membership functions for the categories: HIGH= 0.0883 and NORMAL=0.91169. Next, we create decision rules that will determine how particular levels of association for each of UEMP and BS are combined to establish the levels of association for UE. Fuzzy rules are presented in table 3.

Table 3. Fuzzy rules

Rule	UEMP	BS	UE	Degree
1	VH	VL	VH	1.0
2	VH	L	VH	0.8
3	VH	N	L	1.0
4	VH	H	L	0.8
5	VH	VH	A	0.8
6	H	VL	VH	1.0
7	H	L	H	1.0
8	H	N	H	0.8
9	H	H	A	1.0
10	H	VH	L	1.0
11	N	VL	H	1.0
12	N	L	H	0.8
13	N	N	A	1.0
14	N	H	L	0.8
15	N	VH	L	1.0
16	L	VL	H	1.0
17	L	L	A	1.0
18	L	N	L	0.8
19	L	H	L	1.0
20	L	VH	VL	1.0
21	VL	VL	A	0.8
22	VL	L	L	0.8
23	VL	N	L	1.0
24	VL	H	VL	0.8

25	VL	VH	VL	1.0

This rules are easily human understandable. For example, the first rule

IF UEMP=VL AND BS=VL THE UE=VH

says:

If Unemployment is Very High and number of registered Business Subject is Very Low then the level of Underground Economy is Very High.

The column Degree in table 3 quantifies level of association for the UE. For instance, continuing with 2010 as an example, Rule 8 associates UE with HIGH at a degree of 0.8. This degree suggests that UE is not perfectly associated with HIGH.

The last phase of the analysis involves calculating numeric series for UE. This is achieved by assigning values of 0.0, 0.25, 0.5, 0.75 and 1.0 to the levels Very Low, Low, Normal, High and Very High for UE. Since there are two values for both factors, UEMP and BS, four rules are active for each UE value generated. For example of 2010 the associating values for the four different levels of magnuitude are:

UEMP	HIGH	NORMAL
	0,0883	0,7117
BS	HIGH	NORMAL
	0,9117	0,2883

Now we can form four possible combinations, as follows:

UEMP/BS	**Rule**	**UE level [MIN (UEMP, BS)]**	**UE, MAX**
H/H	9	N: 1*0,0883=0,0883	drop
H/N	8	H:0,8*0,0883=0,0707	
N/H	14	L:0,8*0,7117=0,5694	
N/N	13	N:1*0,2883=0,2883	

The final step is to assign values for the UE levels. These levels have been set to: 0.0, 0.25, 0.5, 0.75 and 1.0, linked respectively to the levels: Very Low, Low, Normal, High and Very High.

For year 2010 we proceed as follows:

Level	**Value**	**Weight**
High	0,0707	0,75
Normal	0,5694	0,5
Low	0,2883	0,25

To obtain resulting numerical value we are conducting defuzzification by using COA (Cener of Area) principle:

$$\frac{0{,}0707 * 0{,}75 + 0{,}5694 * 0{,}5 + 0{,}2883 * 0{,}25}{0{,}0707 + 0{,}5694 + 0{,}2883} =$$

= 0,3657

For the index value for UE to lie in the interval [0,1] the sum of the weights (i.e. the association values) must equal 1.0, which is accomplished by dividing by their sum. E.g. the UE index value of 0.3657 indicates that for 2010 in Croatia the willingness of agents to "go underground" was less than neutral. An average agent, on balance, would tend towards working openly and above-board.

Procedure shown here for 2010 was carried out for all other years. Results are presented in table 4.

Table 4. Estimated index for the underground economy

Year	UE index
2003.	0,9755
2004.	0,8079
2005.	0,7941
2006.	0,4997
2007.	0,2207
2008.	0,2259
2009.	0,2228
2010.	0,3657
2011.	0,5136
2012.	0,5367
2013.	0,4059

Change of theunderground economy index in the last decade is presented at figure 1.

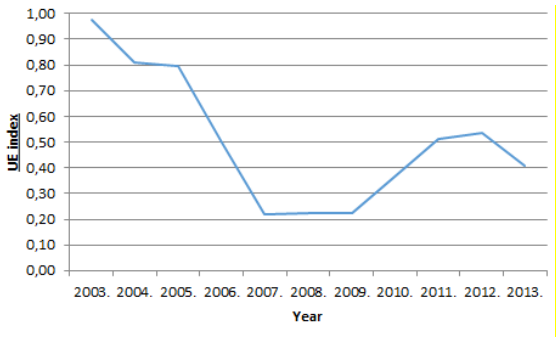

Figure 1. UE index flow through the years

Figure 1 clearly indicates highest level of UE index in the last decade was in 2003. Since 2003 till 2007 level of the underground economy is decreasing. In year 2007 UE index starts to increase till 2012. Last year the level of the underground economy was slightly lower than in 2012.

Reasons behind the largest index in 2003 in the last decade can be found in fact that in 2003 unemployment rate was very high and number of registered business subject was very low. Also, there were none State regulation in the field of unregistered activity. In recent years through state regulation in terms of legislation and the possibility of cheaper business subject`s registration, index of the underground economy was significantly reduced. If the state law have been brought before there might be lower index of underground economy today.

4.1. Unregistered activities

The Act on the Prohibition and Prevention of carrying out unregistered activities [1] defines what is considered to be unregistered activity, who has the authority for combating unregistered activities and supervise the implementation of this law: implementation of administrative measures and criminal legal sanctions

At the time of adoption of this Act supervision over implementation of the same was in the jurisdiction of the State Inspectorate, which is under the Ministry of Finance from 01.01.2014 [6].

Since the implementation of the Act started in the 2011th the question to be asked is: "Who did control unregistered activity in the period until the adoption of the Act and are there differces in the rate of the underground economy before and after the adoption of the Act?" The answer to this question is difficult to give, but we could conclude that until the adoption of the Act , this kind of employment in the economy was not defined and therefore there were no sanctionfor it, which can be seen on the basis of the calculated index of the UE in the initial years of the analysis. If the state regulation began earlier index rate of UE today would probably be much lower. Only one similar regulations which came to the application before than the *Act on the Prohibition and Prevention of carrying out unregistered activity* was *Act on Trade and Commerce* [1,2] which defines *craft* as economic activity of buying and selling goods and/or services and *retailer* as company or person registered for this activity. But, given the fact that this regulation did not clearly define business services, and the method of proofing such activity was difficult.

The table 5 shows the achieved results in the area of identifying unregistered activities by the State Inspectorate in the period of the application of *Act on the Prohibition and Prevention of carrying out unregistered activity* [14,15].

Table 5. Results of inspections unregistered business

Achieved results	2011. th	2012.th
Number of inspections	1095	2163
Number of offenses	318	488
Decision prohibiting	561	369
Measures of seizure of goods	21	22

Based on the data presented in the table we can conclude that the number of controls in unregistered bussines over a period of nearly two years increased one, and the number of identified violations increased too. Adoption of the Act significantly increased the actions aimed at combating the undeground economy. Therefore, we can conlude: if Act was adopted much earlier, UE index would probably be lower, which would have an impact to other negative trends in the economy (unemployment, number of the closed companies, social exclusion, individual decline in GDP).

The analysis of all indicators leads to the conclusion that the state legislation reduces the volume of the underground economy. Results would be better if the same action was conducted much earlier, and the psychological factor of the effects o population would be higher.

5. Measures to reduce the impact of the underground economy

This chapter gives comparative analysis of the methods of registration of a company or bussiness which would improve economic recovery and reduce the UE index. Futhermore, the State should devote greater role to raising awareness of population and encourage them for registration of their activities by financial incentives available to them (the incentives for self-employment), providing cheap education, with the availability of timely and specific information and reducing costs or expenses (registration taxes and costs).

5.1. Registration craft

According to *Crafts Act* [7], the craft is considered to be independent and permanent pursuit of economic activities for the purpose of achieving profit through production, marketing or by providing services on the market. Craftsman is person who performs one or more of the activities in its own name and for its own account. Some activities persons can register as a cottage industry such as services of making jewelry, souvenirs, (services that people do by their own work, or as a secondary occupation activities which includes maintenance and repair).

Homecraft and secondary occupation people can register if they do not achieve revenue of more than ten average gross salaries, at the level of the average gross salary in Croatia.

There are three basic types of crafts [7]:

1. Free crafts which performancedo not require professional qualification exam or master's exam. (for example to perform a hairdressing business people must have passed the master exam, which can be taken after a three years of working experience).
2. Related craft is the types of craft that can be registered by craftsman when he/she has a professional qualification.
3. Preferential craft is a type of craft which requires license or privilege issued by the competent Ministry.

When comparing these types of crafts, related crafts have the biggest obstacles in process of registration. The problem is in the necessary qualifications in

terms of secondary education where often a craftsman who wants to open a business does not have sufficient qualifications. This obstacles lead to growing index of UE.
However, registration of craft gives some advantages in comparison to other forms of registration activities. The benefits of craft are: low cost of registering, there is a possibility of rasing money from giro, maintain business books is simple, craftsman can alone maintain business books, so it is cheaper. Furthermore, value-added tax is paid only when it is actually charged from customers, it is cheaper and easier to modify data such as address, name and crafts activities. When we talk about the disadvantages, it is important to note that a craftman for obligations corresponds alone with all its assets, income tax is paid in accordance with the tax rates on salaries, which are from 12% till 40% there is the impossibility of carrying out all activities because some require appropriate qualifications, as noted earlier.

5.2. Registering of the company

A company is a legal entity. There are the following types of companies: public company, limited partnership, joint stock company, limited liability company, and simply company [18].
To open a company registration at the Commercial Court is required. A company that wants to do business on the Croatian territory must have an address in the Republic of Croatia. The company's name is name on which the company operates.
The following are the advantages of registering a company: the owner or director is responsible for the company up to the amount that is invested (today for simply company it can be just 10,00 kunas). Company pasy the income tax of 20 % for the year, the company, as a legal entity, is independent of the owner, allows registration of any activity, there can be an unlimited number of activities regardless of the education level. However, the real performance of some activities there is a need to hire workers who have the appropriate qualifications.
The main disadvantage of registering the company is a large amount of money for a deposit, it is very expensive and time consuming process of closing the company. Furthermore, value-adde tax needs to be paid at the time when the company issues a bill, no matter if the amount for which the company has released a bill has not yet been paid. Bussiness books maintaing requiress additional costs because they have so called dual accounting. The owner can not raise money from the account without the paper justification. The biggest problem with the registration of any type of activities in Croatia are high costs at the beginning.

The Croatian Government's Guidelines for the implementation of active employment policy in 2014 [12] defines methods and measures for minimization of underground economy index. This should be achieved through a variety of support programs and support for self-employment in the form of irreversible financial resources and politics of connectivity between buyers and producers of services and products.

6 Conclusion

This paper presents the results of fuzzy logic application for estimation the UE in Croatia for the period from 2003 to 2013. The estimated size of the UE index varied from 0,9755 in 2003 to 0,2207 in 2007. Although the level of the underground economy declined in the period from 2003 to 2007, this has not been continued and later given a high level of unemployment. Current economic gap and the high rate of unemployment which is visible even in first quarter of 2014 shows a declining trend and affects a further increase in the volume of the underground economy in the form of performing illegal activities. Measures of state policy aimed to solve this problem are not sufficiently strong. Furthermore there is a huge gap in legislation, since there is a large amount of legislation which leads to misunderstandings and theit unenforceability.

The application of the fuzzy logic approach to the Croatian case and the econometric estimation of the Croatian underground economy profile show that this approach have some appealing features regarding the analysis of underground economy activities. Fuzzy modeling shown to be a powerful tool for solving this problem where information is vague, ill-structured and uncertain. Furthermore, this is a first attempt to deal with underground economy in Croatia from scientific point of view.

References

[1] Act on the Prohibition and Prevention of carrying out unregistered activity (“Narodne novine” 61/13), http://narodne-novine.nn.hr/clanci/sluzbeni/2011_06_61_1349.html, downloaded: April 17th 2014.

[2] Act on Trade and Commerce (“Narodne novine” 87/08,116/08,76/09,114/11,130/11, 68/13 and 30/14), http://narodne-novine.nn.hr/clanci/sluzbeni/340389.html, downloaded: April 25th 2014.

[3] Bejaković, P. Macroeconomic measurement of the informal economy, Financial Practice, 21 (1-2), 71-91, 1997.

[4] Bordignon, M.; Zanardi, A., Tax evasion in Italy, G. Econ. Ann. Econ. 56, 121–169, 1997.

[5] Breusch, T. The Canadian Underground Economy: An Examination of Giles and Tedds, Canadian Tax Journal, 53 (2), 367–91, 2005.

[6] Customs Service Act ("Narodne novine" 68/13 and 30/14), http://narodne-novine.nn.hr/clanci/sluzbeni/2013_06_68_1346.html, downloaded: April 6th 2014.

[7] Crafts Act ("Narodne novine" 143/13), http://narodne-novine.nn.hr/clanci/sluzbeni/305213.html, downloaded: April 17th 2014.

[8] Draeseke, R; Giles, D.E.A. A fuzzy logic approach to modeling the New Zealand underground economy, Math. Comput. Simulat. 5 115–123, 2002.

[9] Eilat, Y.; Zinnes, C. The evolution of the shadow economy in transition countries: consequences for economic growth and donor assistance. CAER II Discussion Paper, 83, Harvard Institute for International Development, 2000.

[10] Feige, E. L. Defining and estimating underground and informal economies: The new institutional economics approach, World Development, Elsevier, vol. 18(7): 989-1002, 1990.

[11] Giles, D.E.A.; Tedds, L.M. Taxes and the Canadian Underground Economy, Canadian Tax Foundation, Toronto, 2002.

[12] Guidelines for the implementation of active employment policy 2014, Ministry of Labour and Pension System, https://vlada.gov.hr/UserDocsImages//Sjednice/Arhiva//126.%20-%2010.pdf, downloaded: March 18th 2014.

[13] Lovrinčević, Ž; Marić, Z; Mikulić, D. Maastricht criteria, the underground economy-case Croatia, 2006.

[14] State Inspectorate Report for 2011., downloaded: http://www.inspektorat.hr/Upload/Documents/Godisnje-izvjesce/20120613_DIRH-Godisnje%20izvjesce%202011.pdf, last accessed: 02.05.2014.

[15] State Inspectorate Report for 2012., available at: http://www.inspektorat.hr/Upload/Documents/Izvjesca/20130527_Izvjesce.pdf, last accessed: 01.05.2014.

[16] Tanzi, V. The underground economy, Financ. Dev. 20 (4), 10–13, 1983.

[17] Topić, M. Underground Economy in Croatia, Croatia revision, 100-103, 2006.

[18] The Comapnies Act ("Narodne novine" 152/11,11/12 and 68/13), http://narodne-novine.nn.hr/clanci/sluzbeni/2011_12_152_3144.html, downloaded: April 17th 2014.

[19] Yu, T.H.K.; Han-Min, D.; Chen, S.J. A fuzzy logic approach to modeling the underground economy in Taiwan, Physica A 362:471-479, 2006.

[20] Zadeh, L.A. Fuzzy logic, neural networks, and software computing, Commun. ACM 37 (3): 77–84, 1994.

Appendix

Table 6. Membership functions

$\mu_{VL}(x) = 1.0$	$if\ x \leq Mean - 2SD$
$\mu_{VL}(x) = \frac{Mean - SD - x}{SD}$	$if\ Mean - 2SD \leq x \leq Mean - SD$
$\mu_{L}(x) = \frac{x - Mean + 2SD}{SD}$	$if\ Mean - 2SD \leq x \leq Mean - SD$
$\mu_{L}(x) = \frac{Mean - x}{SD}$	$if\ Mean - SD \leq x \leq Mean$
$\mu_{N}(x) = \frac{x - Mean + SD}{SD}$	$if\ Mean - SD \leq x \leq Mean$
$\mu_{N}(x) = \frac{Mean + SD - x}{SD}$	$if\ Mean \leq x \leq Mean + SD$
$\mu_{H}(x) = \frac{x - Mean}{SD}$	$if\ Mean \leq x \leq Mean + SD$
$\mu_{H}(x) = \frac{Mean + 2SD - x}{SD}$	$if\ Mean + SD \leq x \leq Mean + 2SD$
$\mu_{VH}(x) = \frac{x - Mean - SD}{SD}$	$if\ Mean + SD \leq x \leq Mean + 2SD$
$\mu_{VH}(x) = 1.0$	$if\ x \geq Mean + 2SD$

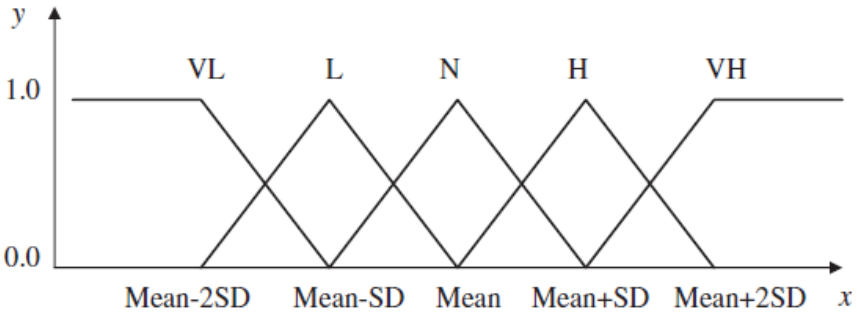

Figure 2. Membership function