## Resource Constrained Project Scheduling under Uncertainty: A Survey

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**Abstract.** Resource Constrained Project Scheduling Problems (RCPSP), especially their stochastic variants, and the methods operating on them represent a general project scheduling optimization framework.

This paper presents the survey of methods and models that are put into the historical context and are categorized according to their working principles. It aims to supplement and update existing RCPSP surveys. The current state of the research field is assessed and potential research venues are identified.

Keywords. project scheduling, resource constraints, uncertainty

## 1 Introduction

Human society has never before been so technologically advanced. Ever more sophisticated and complex systems are incessantly being developed through organized processes called projects, which tackle the ever-growing complexity.

Definition of project as given in ISO 9000:2005 is: "Project - unique process, consisting of a set of coordinated and controlled activities with start and finish dates, undertaken to achieve an objective conforming to specific requirements, including the constraints of time, cost and resources."[1]

Through the lifecycle of a project, management is mostly involved in planning, scheduling and control. This paper addresses the scheduling and control phases of project lifecycle, assuming the project plan already given as output of the planning phase.

The importance of project scheduling and control is bolstered with many examples of late and over budget projects. Inadequate scheduling and control are often identified as the most common causes of project failure [2]. Standish Group survey [3] reports that in 2009 only 32% of IT projects were successful, 24% failed and 44% were either over budget, late or with unfulfilled expectations.

Several surveys related to RCPSP have been published in the past. Deterministic variants and the methods for solving them are surveyed in [4]. Enumeration of deterministic variants and extension of the basic model is given in [5] but with the solution methods left out. Heuristic methods for the basic variant were surveyed in [6]. An influential manuscript [7] deals with stochastic models and methods for both machine and project scheduling and [8] offers its update for the project scheduling. Review of robust and reactive scheduling procedures is given in [9]. Finally, surveys of stochastic machine scheduling problems such as [10] are important because of the natural transfer of ideas from that field to the project scheduling. Our aim is to supplement and update previous works but also at the same time keep the self-containment of the presentation.

This paper is organized as follows: short history of project scheduling is covered in section 2. Section 3 presents Resource Constrained Project Scheduling Problem, its variants, solving procedures with the emphasis on stochastic variants. We conclude the paper in the section 4.

# 2 Short history of project scheduling

Formal research in problems of project scheduling and planning started after the World War 2 and here are presented some of the most important modern discoveries in project scheduling research.

Critical path method (CPM) and Program Evaluation and Review Technique (PERT) emerged about the same time in late 1950s. Both are activity network methods using activity-on-node representation, both assuming unbounded availability of resources. CPM assumes deterministic durations of activities and PERT incorporates uncertainty in activity durations. The main problem with both methods is the omission of resource constraints. Also, uncertain activity durations are not the only sources of unpredictability in real world projects. PERT -like models are, however, used for representing time-cost tradeoff problems.

**Graphical Evaluation and Review Technique** (**GERT**) appeared in mid 1960s. It is a stochastic activity network method similar to PERT but enables more general descriptions in project graphs, also involving project network structural uncertainty which affects the path of traversal of the project graph. The problem with this model is the assumption of unlimited resources.

**Resource Constrained Project Scheduling Problem (RCPSP)** [11] is a deterministic problem which provides an extension to the CPM problem adding resource usage constraints. RCPSP came to the focus of the scientific research in 1980s and remained there during 1990s. The limiting factor of this model is in its deterministic nature.

**Critical Chain Project Management (CCPM)** was presented in 1997 in [12]. This was one of the first proactive methods that accounted for uncertainty in RCPSP, creating schedules protected with temporal buffers. Criticism for this method is given in [13] with main objection that its main concern is feasibility and not optimality. This method can be characterized as proactive heuristic algorithm solving SRCPSP.

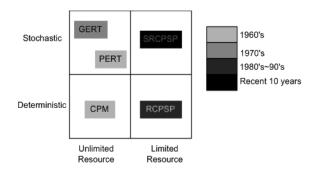


Figure 1. Evolution of project scheduling [14]

**Stochastic Resource Constrained Project Scheduling Problem (SRCPSP)** is a stochastic variant of the RCPSP and it can involve many sources of uncertainty like: activity durations, renewable resource availability, task insertion, resource consumption etc. The interest in SRCPSP was spurred in the last 10 years.

This paper focuses on deterministic and stochastic RCPSP and they are discussed and described to more detail in the rest of the paper.

## 3 RCPSP

RCPSP searches for the minimal makespan nonpreemptive schedule subject to activity precedence and resource availability constraints. It is defined as a combinatorial optimization problem and it has many variants. Bruckner et al. [4] introduced a classification scheme compatible with machine scheduling which unambiguously describes resource environment, the activity characteristics and objective function.

Formally the basic variant of RCPSP is combinatorial optimization problem defined by the tuple (*V*, *E*, *p*, *R*, *B*, *d*, *f*) where; we are given a set of n+2 activities  $V = \{A_0, ..., A_{n+1}\}$  where  $A_0$  and  $A_{n+1}$  are, by convention, dummy activities that represent the start and the end of the schedule. Precedence relation is defined as transitive closure of the relation  $E \subset V \times V$  with a constraint on E such that A<sub>0</sub> precedes all activities and  $A_{n+1}$  is a successor to all activities. We use activity-on-node representation of the project, defined by directed acyclic graph G(V,E). Activity durations are represented by the vector  $p \in \mathbb{N}^{n+2}$  where  $p_i$  marks the duration of activity  $A_i$ with  $p_0 = p_{n+1} = 0$ . We are also given a set of *renewable* resources  $R = \{R_1, ..., R_r\}$  and their availabilities  $B \in \mathbb{N}^r$ . Activities need one or more resources for processing and those demands are captured in  $d \in \mathbb{N}^{(n+2) \times r}$ , where dummy activities have no demands for resources. Last element of the tuple is the objective function  $f: \mathcal{X} \to \mathbb{R}$ , which maps from the feasible schedule space to the real values. Usually this last member of the tuple is omitted as it is assumed to be just the makespan of the schedule, unless stated otherwise.

Solutions to RCPSP are precedence and resource feasible schedules  $S \in \mathbb{N}^{n+2}$  where  $S_i$  represents the start time of activity  $A_i$ . We assume that  $S_0=0$  and in that case  $S_{n+1}$  is the makespan of a schedule. Completion times of activities are represented by C = S + p. Precedence constraints are:

$$(S_j \ge S_i + p_i), \forall (A_i, A_j) \in E$$
 (1)

In order to define resource constraints, we introduce the set of concurrent activities at a time t as:

$$\Lambda_t = \{ A_i \in A \mid S_i \le t \le C_i \}$$
(2)

Then resource constraints are given with:

$$\sum_{A_i \in \Lambda_t} d_{ij} \leq B_j, \forall t \geq 0, \forall R_j \in R$$
(3)

RCPSP is a general and powerful scheduling model and it contains various machine scheduling problems such as job shop problem and flow shop problem as special cases.

RCPSP is NP-hard in the strong sense [15].

#### 3.1 Variants

There are many deterministic extensions/variants of the RCPSP. We refer the reader to [4] and [5] for more detailed description. In this paper we just mention the most common ones with the focus on two special variants mentioned.

Possible extensions to the model define changes in the assumptions of the basic model. The model can be extended to use nonrenewable and partially renewable resources. The former can model the budget, available only in a predetermined amount during the whole project. The latter is a combination of renewable and nonrenewable resources, as they renew only with predetermined period. Partially renewable resources can be used to model various legal constraints. Objectives can be changed to the ones based on: Net Present Value (NPV), cost, robustness, renewable resources (resource renting related), nonrenewable resources, rescheduling costs or even multiple objectives. Time constraints can include minimal and maximal time lags between activities and release dates. We can also broaden our project scheduling to the multi-project setting. Time-cost trade-off variant is a setting with a nonrenewable resource where we have options to lower the durations of activities at the expense of the budget (nonrenewable resource).

**Multi-Mode RCPSP (MM-RCPSP)** permits activities to be processed in one of several modes. Each of the modes has different resource requirements and processing time and resources used can be renewable and nonrenewable.

Multi-Skill Project Scheduling Problem (MSPSP) adds the possibility of using a resource in more than one manner. In this setting, resources are renewable and disjunctive, representing staff members that have more than one skill and it is assumed that each activity needs some skills in certain amount to be performed.

#### 3.2 Solving deterministic RCPSP

As we are dealing with NP-hard problems, exact solving is possible only for small and sometimes medium sized problems. For big instances we have to resort to metaheuristics.

#### 3.2.1 Exact procedures

The setting of a problem searches for schedules with integer activity start times [11].

One of the best and most commonly used methods for finding exact solutions is the branch-and-bound (B&B), which manages to solve some medium sized instances. Upper bound is obtained using heuristics, while lower bounds can be found using Mathematical Programming ([16]) or using various precedence and resource relaxations of the problem which tend to be faster but produce weaker bounds [17–19].

Mixed Integer Linear Programming (MILP) can be used to model RCPSP and be solved either using branch-and-bound, branch-and-cut, branch-and-price or some other method. Koné et al. created an efficient RCPSP MILP formulation in [20], which is also used as modeling inspiration for other solving methods.

Horbach in [21] transformed RCPSP to satisfiability problem and used Satisfiability Testing (SAT) solver for best results at that time.

Constraint Programming (CP) is a method that solves RCPSP with constraint-based scheduling. Schutt at al. in [22] use a combination of CP with SAT in algorithm that has proved to be very efficient and, at the time of writing, performed as the best on PSPLIB test set. Satisfiability Modulo Theories (SMT) approach, a generalization of SAT, has been used for RCPSP in [23] and proved to be efficient and scalable.

The three latter approaches have solved some of the long standing unsolved test set cases and proved to be promising future research venues.

#### 3.2.2 Heuristics

Heuristic approaches dominate the research on solving RCPSP and its variants.

The simplest methods for approximate solving RCPSP are *constructive heuristics* that use a priority list, the result of used priority rule for activities, as a guide to schedule creation. The most basic members of this family are serial and parallel scheduling schemes. Single-pass methods are very fast, even for huge projects but they don't tend to produce solutions of high quality. These methods are used in popular planning software, such as MS Project. In [24] authors conducted testing which has shown that popular software packages calculate schedules in average 9.27% worse than the best known solutions.

Double justification [25] is a very promising heuristics that can improve the initial solution and is especially good when embedded in other heuristics, as was demonstrated in the paper on the example of genetic algorithms and simulated annealing.

Metaheuristics are improvement heuristics, a form of stochastic search, they tend to produce very good solutions, reasonably close to the optimum. Decomposition based genetic algorithm was used in [26] for solving RCPSP which resulted in some of the best results on standard benchmarks. Bi-population genetic algorithm, with the mode improvement procedure, used in [27] for solving MM-RCPSP proved to be very competitive on benchmarks. Hybrid algorithm using a combination of ant colony optimization and scatter search has been used in [28] to obtain very good results on RCPSP. Many other methods were applied as well, such as: simulated annealing, electromagnetism metaheuristics, taboo search, etc. Hartmann and Kolisch surveyed state-ofthe-art heuristics in [6] and came to the conclusion that the best performing methods are metaheuristics that use population-based approaches. They also pointed at the usefulness of the local search in the form of double justification.

Lately, there has been a raised interest in combining learning mechanisms from machine learning with heuristics and metaheuristics. Wauters et al. in [29] used multi-agent learning automata approach for solving MMRCPSP. Authors in [30] used A-Team multi-agent architecture for tackling the MMRCPSP/Max as well as other RCPSP variants in their previous works.

#### 3.3 SRCPSP

SRCPSP is a stochastic variant of the RCPSP that proves to be more adequate modeling platform. Real world projects' parameters are prone to errors in estimation and external unexpected influence. Both causes can result in deterministic schedules being longer and more expensive than expected or even becoming infeasible.

SRCPSP introduces mainly non-structural uncertainty into the basic deterministic model. Possible sources of uncertainty can be: duration of activities, resource consumption, resource availability, stochastic task insertion etc. Much research in SRCPSP so far has focused on stochastic activity durations [8].

#### 3.3.1 Solving procedures

There are three main approaches for solving SRCPSP: predictive, proactive and reactive approaches.

#### 3.3.1.1 Predictive methods

*Predictive methods* approach to the SRCPSP ignoring the uncertainty and proceed to finding the solution using methods for the deterministic variant. These methods are used for creation of the baseline schedule which can be used for planning external activities, the ones connected to the entities indirectly included into the project. Such activities are procurement, maintenance and delivery [31]. It is a long known fact that (for example from [32]) substituting random durations with their expected values results in an makespan underestimation. Predictive schedules, affected by uncertainty, can be late, over the budget or even become infeasible. Proactive schedules make for better baseline schedules and, as such, serve as better communication basis between project entities.

#### 3.3.1.2 Proactive methods

*Proactive methods* try to cope with uncertainty by increasing the robustness of the schedule to unexpected outcomes so that the schedule can remain feasible under various conditions and functional in uncertain environment. Most of published works were in the field of machine scheduling. Surveys of proactive methods, in both machine and project scheduling, are given in: [8], [7], [10], [9]. Possible approaches are [8]: redundancy based methods, robust scheduling methods and contingent scheduling.

*Redundancy based methods* achieve robustness to perturbations through resource and time redundancy. Time redundancy is more frequently used because the resource redundancy is often prohibitive due to the costs. Temporal protection extends activity durations or inserts time buffers based on the statistics of used resources. In [33] to each activity is given an allowed slack, while [34] uses heuristics to minimize maximal lateness due to resource breakdowns. Critical Chain Project Management, described in part 2, is a representative of these methods. SRCPSP with stochastic task insertion and proactive scheduling methods using buffers have been in the focus of [35].

The goal of *robust methods* is minimization of the cost function such as the weighted sum of expected deviations between the baseline and realized schedule. [36] uses specially defined robustness measure, abstraction of resource usage and linear programming. Resource redundancy was the focus of [37] where the authors look for resource allocation algorithms that would increase the schedule stability. They have proposed a method based on search for optimal resource flow network using B&B. Time redundancy is achieved with insertion of time buffers in order to prevent the propagation of disturbances through the schedule. Exact and heuristic time buffer insertion methods, including reasonably effective STC heuristic algorithm, are developed in [38-42], as well as reactive methods for robust solutions. Resource availability is yet another research venue. Authors in [43] use proactive-reactive approach to solve the problem of uncertainty in renewable resource availability. Eight proactive and three reactive procedures were defined. Deblaere et al. in [44] created integer programming based heuristics which maximizes schedule stability under uncertainty of activity durations.

Proactive scheduling in MMRCPSP with stochastic nonrenewable resource usage is the focus of [45], which minimizes the resource duration and guarantees a certain probability to satisfying nonrenewable resource constraints.

*Contingent scheduling* is the approach that might be considered both proactive and reactive. Disruptions are *a priori* predicted and a number of schedules is created, before or at the runtime, which are used as countermeasures for expected disruption events. This approach is "focused on flexibility, rather than robustness, and is especially valuable for time-critical reactive scheduling"[8].

#### 3.3.1.3 Reactive methods

*Reactive methods* contain two subgroups of methods. Baseline schedule repair methods constitute the first subgroup and the dynamic scheduling methods, which work completely online, constitute the second one. The latter methods don't use baseline schedule but see the project as multistage decision-making process and dynamically create a schedule in stages at runtime using policies.

Baseline schedule repair reactive methods are called predictive-reactive or proactive-reactive, depending on the used baseline plan. Predictivereactive methods are efficient when there is no stochastic model of possible disruptions or the degree of uncertainty is too great. The simplest representatives are techniques for quick restoration of schedule feasibility; simple schedule repair actions. The most elementary repair action is the right shift *rule*, a heuristic algorithm that tends not to perform well because it does not change the order of actions in schedule, which is something that might be needed for better solutions. Complex methods seek for complete rescheduling. The goal of total re-optimization is most often dropped due to the NP-hardiness of the problem that makes it too time-consuming. We also want to avoid significant differences between prior and posterior schedules. The usual assumption is that the original schedule is a good starting point, therefore many reactive methods try to create a new schedule that minimally deviates from the original schedule, using perturbation measures. Weighted sum of absolute differences between starting times of activities in the original and the new schedule is a frequently used example of measure for perturbation. [46] describes the linear integer model for MMRCPSP that is solved using combined MILP/CP approach in order to minimize the deviation between schedules caused by disturbance in duration of one activity. The choice of rescheduling point is also a feature of the algorithm. Possible choices are: with every disturbance [31], in pre-set intervals or moments [47], or with exceeding some threshold of the measure of robustness of schedule [48]. In [49] the authors used stepwise heuristic algorithm on the resource flow network for stochastic activity insertion in multimodal environment while preserving prior resource allocations. Rout-algorithm from machine learning was used in [50] for project schedule repair. Constraint programming approaches use problem of dynamic constraint satisfaction for SRCPSP upon which various proactive and reactive solutions have developed [51]. Some reactive solutions are based on intelligent backtracking, usage of old solution and usage of reasoning [11]. Explanation-based Constraint Programming (e-CP) uses past reasoning for routing the search into relevant parts of the search space that performs better than classical backtracking. Elkhyari, in [52] and [53], applied e-CP for relatively efficient predictive-reactive procedure.

The work on dynamic reactive scheduling (called stochastic project scheduling in [8]) started with the research on various scheduling policy families, with emphasis on stable policy families. [54] works on early start and preselective policies, [55] and [56] added priority variants and [57] defined linear preselective policies. Resource-based policy families are based on finding minimal forbidden sets of activities whose number can grow exponentially in number of activities which is the main flaw of search for optimal solutions in these families. Solving algorithms for these problems are given in [58]. Stork presented B&B exact search methods in [59] for optimal policies in various families. Ashtiani et al. defined pre-processing policy family in [60] and they searched within the family using two-phase genetic algorithm. PP-policies make some decisions before the project start and the rest at the runtime. Good members of this family outperform optimal early start policies. Deblaere et al. presented resource based policies with release times in [61]. The search for optimal solution is implemented in three-phase neighborhood search. Lambrechts implemented in [62] moderate performing reactive algorithm using taboo search that tries to protect the new schedule from future disruptions. Artigues et al. in [63] used robust optimization to find an early start policy that minimizes maximum absolute regret in problems with unknown probability distributions of activity durations. As the exact procedure proved to be intractable, heuristic procedure was developed. Padberg introduced in [64] the description of software project as a Markov Decision Process, described diverse model construction procedures and used policy iteration in order to find the optimal policy. The complexity of this procedure is exponential. Li, in [65], uses rollout algorithms combined with constraint programming with promising results. Choi et al. in [66] tackled a complex SRCPSP with dynamic programming, combining it with a heuristic that confines the state space. In [67] the authors used Qlearning on similar, but multiproject, problems to alleviate the difficulties of manipulating the analytical transition model, encountered in the first paper. Csaji and Monostori [68] utilized Q-learning on stochastic resource allocation problems. They presented the effectiveness of their approach on benchmarks and on an industry related test, all of them being instances of machine scheduling problems.

Drezet and Billaut ([69]) describe the application of the special variant SRCPSP in IT company. Three approaches were used: predictive for creation of the primary plan, proactive for stabilization of the primary plan resulting in the baseline plan and finally reactive for schedule repair.

## 4 Conclusion

Resource Constrained Project Scheduling is a well researched problem, but the research in scheduling under uncertainty has been intensified only in the last 10 years. As, in most of variants, we are dealing with a NP-hard problem, approximate methods have to be used for large instances. Although research studies single out dominant heuristic approaches, none of the methods has shown to be the best. Most of the research up to date has dealt with only one source of uncertainty, mostly in duration of activities, and has focused mainly on the basic RCPSP model. Methods dealing with uncertainty are proactive or reactive. Proactive methods work better in cases when the uncertainty is quantifiable while in other cases, or when the degree of uncertainty is too great, reactive methods perform better [7]. Authors in [8] and [11, 211] indicate the perspective of research in combining proactive and reactive methods. The necessity for new models of scheduling which account for production environment conditions, like for example, data flows, their interpretation and understanding of interaction between the main schedule components is asserted in [70].

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Potential for further research lies in multisource uncertainty models, generalized RCPSP variants and solutions based on combined proactive-reactive approaches. Also, the research could look into the potential for creation of reactive methods with robust solutions as well as possible extensions of project model and the necessity for them.

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