

Some traffic control proposals by means of fuzzy sets theory

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Abstract. *The paper introduces a method of vehicles' stop time prediction on an intersection entrance. The method allows us applying an adaptive traffic control algorithms [1], covering the transportation network with intersections, not provided with traffic detectors or some of the detectors are not working properly. The input data is taken from the detectors set placed at the adjacent intersections. The vehicles' positions is assigned by means of the data fusion [2], assigning the initial position of vehicles, as a fuzzy number [3]. The simulation process combines a fuzzy cellular automaton [5], [6], [7], [8], [9] based on the Nagel-Shrekenberg [4] model. The vehicles' positions, length, velocity, maximal velocity and acceleration were defined differently – using fuzzy numbers. The innovation of the discussed method concerns a specific manner of vehicles' overtaking maneuver modeling. Unlike an earlier presented models, where overtaking was not considered or was significantly simplified. The method takes into account the simulation process of traffic states on neighboring traffic lanes.*

Keywords. traffic control, fuzzy sets, cellular automata, vehicle overtaking modeling

1 Introduction

The traffic adaptive control algorithms work on vehicles' volume, recorded on traffic lanes. The needed data assigns the vehicles' number and time schedule of their rush within the stop lines of the intersection inlets. The fully adaptive control processes have to be provided with traffic detectors in specified points. Anyhow, one can find places in the traffic model, where the detectors are not working

properly or they are not needed at all. The question that has to cope with, the controlling procedures developer concerns, what to do with this not completed information system.

The so called microscopic traffic model allows us predict the traffic in specific places, in spite the direct measures are not available (with missing detectors' points). The traffic modeling with more general vehicles' flow analysis can provide us with the missing data. That is why the process of traffic control will still be working with satisfactory quality. The needed input data, for simulation algorithm can be found within detectors at adjacent intersections. The multi-sensors' data fusion can determine the vehicles' parameters. such as position, velocity, etc.

The uncertainty of the vehicle's state was assigned by means of fuzzy numbers, briefly described below

2 The fuzzy sets and fuzzy numbers theory

The fuzzy logic and fuzzy numbers are used for defining the uncertainty of data expressions, used further for the traffic description.

2.1 The fuzzy logic and sets

The fuzzy set of elements, of certain not empty space X , is called the set of ordered pairs:

$$A = \{(x, \mu_A(x)) : x \in X\}$$

Where the function $\mu_A : X \rightarrow [0; 1]$ is a membership function of a fuzzy set A . This function allows us assessing the membership level of the number $x \in X$ to the fuzzy set A .

The subset of the membership function's arguments, which value is greater than 0, is called a support of a fuzzy set A .

$$supp(A) = \{x \in X: \mu(x) > 0\} \quad (1)$$

The maximal value of the membership function is called height of the fuzzy set A:

$$h(A) = \max_{x \in X} \{\mu(x)\} \quad (2)$$

When the height $h(A)$ of fuzzy set A equals 1 then A is called normalised.

The subset of the space X, which elements' membership function values are equal to 1 define a core of the fuzzy set A

$$core(A) = \{x \in X : \mu(x) = 1\} \quad (3)$$

The fuzzy set A is convex if the following inequality relation is true:

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\} \quad (4)$$

for all $x_1, x_2 \in X$ and $\lambda \in [0; 1]$.

In Fig. 1 the membership function diagram of a hypothetical fuzzy set is introduced, where $X = R$. The Fig. 1. illustrates all the terms defined above.

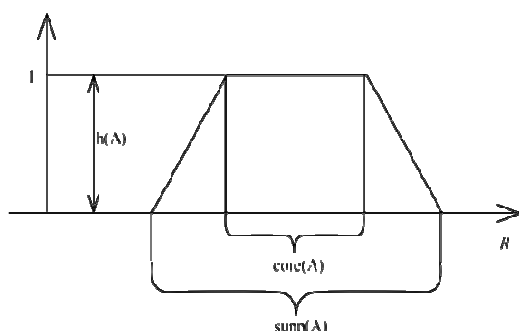


Fig. 1 The fuzzy set's membership function diagram

The arithmetic operations on the fuzzy numbers are defined by means of the functions called T-norms and T-conorms (or triangular norms).

A function: $T : [0; 1] \times [0; 1] \rightarrow [0; 1]$, is called a T-norm, when the following conditions are fulfilled for all $a, b, c, d \in [0; 1]$.

The function T

- is non-decreasing with respect to both arguments:

$$T(a, c) \leq T(b, d), \text{ where: } a \leq b, c \leq d$$

- is commutative, for:

$$T(a, b) = T(b, a)$$

- is associative, for:

$$T(a, T(b, c)) = T(T(a, b), c)$$

- fulfills a border condition, for :

$$T(a, 1) = a$$

A function $S : [0; 1] \times [0; 1] \rightarrow [0; 1]$, is called a T-conorm, when the following conditions are fulfilled, for all $a, b, c, d \in [0; 1]$ function S:

- non-decreasing with respect to both arguments:

$$S(a, c) \leq S(b, d), \text{ where } a \leq b, c \leq d$$

- is commutative, for:

$$S(a, b) = S(b, a)$$

- is associative, for:

$$S(a, S(b, c)) = S(S(a, b), c)$$

- fulfills the border condition:

$$S(a, 0) = a$$

The membership function's value of a sum of fuzzy numbers is expressed by:

$$\mu_{A \cup B}(x) = S(\mu_A(x), \mu_B(x)) \quad (5)$$

Similarly the membership function value of a product of the fuzzy numbers is defined by formula:

$$\mu_{A \cap B}(x) = T(\mu_A(x), \mu_B(x)) \quad (6)$$

The simplest example of triangular norms are minimum and maximum functions:

$$T(a, b) = \min\{a, b\} \quad (7)$$

$$S(a, b) = \max\{a, b\} \quad (8)$$

For the model introduced in the paper the product T-norm has been defined as:

$$T(a, b) = a \cdot b \quad (9)$$

2.2 The fuzzy numbers

The fuzzy number is a specific case at the fuzzy set, with real numbers domain. The fuzzy set A can also be called a fuzzy numbers set if it fulfills some conditions, like:

- is normalized, $h(A) = 1$,
- is convex,
- its support is a range,
- its membership function μ_A is continuous,
- its core consists of only one element.

On the basis of foregoing definition one can see the set, which membership function was introduced in Fig. 1. It is not a fuzzy number because its core is a range therefore the last condition is not fulfilled.

The model introduced in this paper works on the strength of fuzzy cellular automaton [5]. The modeling parameters, like vehicles' position and velocity, are represented by the discrete values of variables. The all discussed parameters are expressed by the fuzzy numbers.

That is why the defined above fuzzy numbers understanding will not be suitable for the cellular automata assignment. For the cellular automata

description improvement, some discrete fuzzy numbers have been introduced. The example of these fuzzy numbers is presented in Fig. 2.

Some arithmetic operations of continuous fuzzy numbers were defined below:

- the membership function of the sum of fuzzy numbers $B = A_1 + A_2$:

$$\mu_B(y) = \sup_{\substack{x_1, x_2 \\ y = x_1 + x_2}} \{ \min\{ \mu_{A_1}(x_1), \mu_{A_2}(x_2) \} \} \quad (10)$$

- the membership function of the difference of fuzzy numbers $B = A_1 - A_2$:

$$\mu_B(y) = \sup_{\substack{x_1, x_2 \\ y = x_1 - x_2}} \{ \min\{ \mu_{A_1}(x_1), \mu_{A_2}(x_2) \} \} \quad (11)$$

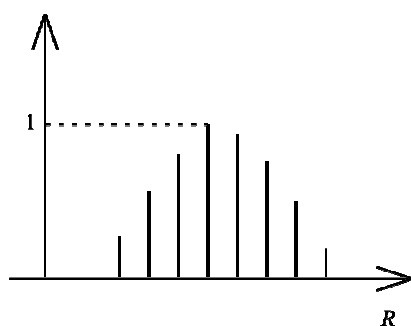


Fig. 2 Membership function diagram of a discrete fuzzy number

The above operations are turned out not sufficient for the discrete fuzzy numbers. The membership function obtained as a result, appears excessively irregular. That is why above limits will not be suitable for an implementation of the described phenomenon properly. For the above operations, refining some modifications that were introduced earlier, is undoubtedly needed.

Instead of the minimum T -norm, like it assign equations (10) and (11), the product T -norm has been used.

After these changes the arithmetic operations on the discrete fuzzy numbers, were defined as follows:

- the membership function of the sum of discrete fuzzy numbers $B = A_1 + A_2$:

$$\mu_B(y) = \max_{\substack{x_1, x_2 \\ y = x_1 + x_2}} \{ \mu_{A_1}(x_1) \cdot \mu_{A_2}(x_2) \} \quad (12)$$

- the membership function of the difference of discrete fuzzy numbers $B = A_1 - A_2$:

$$\mu_B(y) = \max_{y = \min\{x_1, x_2\}} \{ \mu_{A_1}(x_1) \cdot \mu_{A_2}(x_2) \} \quad (13)$$

3 The fuzzy cellular automata

For the simulation process the Nagel-Schreckenberg cellular automaton has been applied [4]. However a several changes have been made in order to take into account the uncertainty of a vehicle's state parameters and make the model more similar to real traffic conditions.

The most important difference concerns descriptions of all vehicles' parameters. They were expressed by the fuzzy numbers. The arithmetic operation on the fuzzy numbers, in this model, are performed in accordance to the defined previously formulas. The road stretch, under the analysis, is divided into the cells chain. The cells define specific measures of the traffic lane, in a crossroad zone. Each cell unit is equal to the same length.

The set of the cells assigns the domain of the discrete membership function, of fuzzy numbers. These numbers describe the vehicle's positions in the crossroad entrance, for each traffic lane.

There can happen states that in one cell a several vehicles have membership function value greater than zero, but it does not mean that vanguard of more than one car can be found in this particular cell.

As every vehicle's parameter is defined within the fuzzy numbers representation it was convenient assuming that the domain of its membership functions are the same – set of integers. A co-domain of the membership function is bracket $[0, 1]$. Thus the arithmetic operations can be performed straightforward as it was discussed in previous chapter.

The second difference is such as, the vehicles overtaking manoeuvres were also taken into account. This assumptions involves the traffic simulation principles concerning each lane, with both traffic directions at any moment.

The vehicles in the simulation model belong to one of two classes that are differ each other by maximal velocity (3 and 5 cells per simulation step). The faster vehicle can overtake the slower one, on certain conditions: the cell before blocking vehicle must be free and the sufficient number of cells ahead in the opposite direction lane must be free as well.

In Fig. 3 the algorithm part of vehicle's position calculation on the road stretch was introduced. The algorithm makes repetitions for all vehicles moving on the road. Apart from typical position calculations, which are the same as in Nagel-Schreckenberg approach, one can see the overtaking vehicles considered in this model successfully.

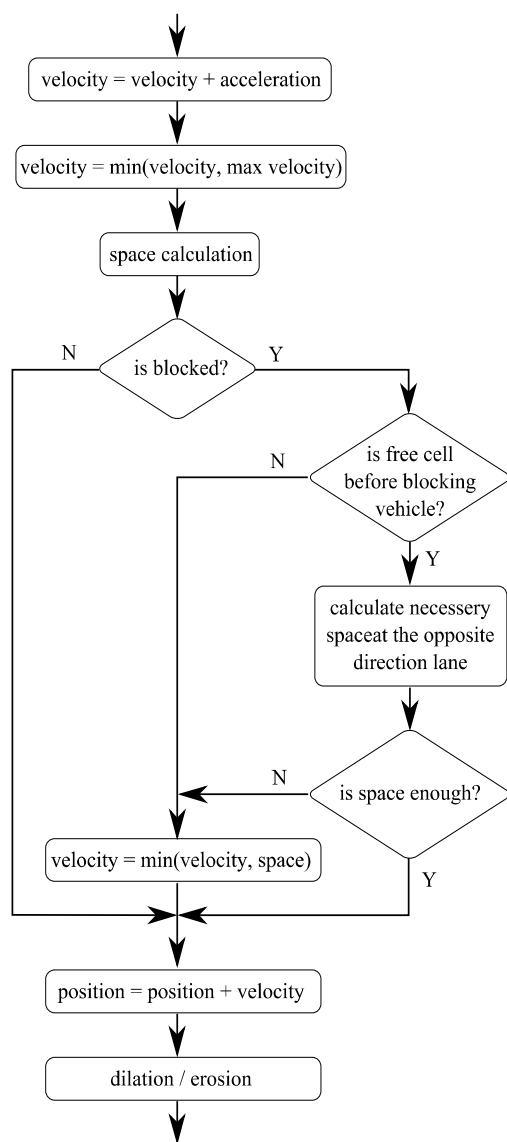


Fig. 3. The vehicle's positions calculation algorithm flow chart

At each step of simulation procedures based on the Nagel-Schreckenberg model, a novel velocity analysis were provided. They were based on the velocity of the previous calculations step, completed by the vehicle's acceleration. The maximal velocity, defining the distance to the preceding vehicle and the preceding vehicle's length are also taken into account. The speeding vehicles description, for every object on the road stretch, allow us the new position for each of them precisely designate.

Some more differences between fuzzy and traditional traffic modeling can also be indicated. The random factor describing probability of velocity reduction was removed from the model. Instead of this two functions: dilation and erosion were introduced.

Dilation increases the blur vehicle position. Increasing of the blur occurs when the vehicle is accelerating or running at top speed. The vehicle's position on a road section is becoming increasingly vague.

Erosion causes the sharpening of the vehicle position. Sharpening the vehicle's position occurs when its velocity decreases or is constant but smaller than the maximal velocity. This is justified by the fact that the vehicle position is unambiguously determined by the other traffic participants and it is blocked by them.

Dilation and erosion functions operate differently for the "front" (for arguments greater than that for which the value of membership function is equal to 1 (equation 2)) and the "rear" (equation 3) part of the membership function of vehicle's position. This division allows to take into account the membership function that becomes asymmetric, due to the blocking vehicle by other traffic participants. Thus, in every step of the simulation, the fuzzy-number defining the vehicle's position is modified as follows:

$$\forall_{\substack{x \in D \\ x > \hat{P}}} \mu_{p'}(x) = [\mu_p(x)]^\alpha \quad (14)$$

$$\forall_{\substack{x \in D \\ x < \hat{P}}} \mu_{p'}(x) = [\mu_p(x)]^\beta \quad (15)$$

where:

- D - set of cells of the automaton
- α, β - function parameters (for dilation and erosion)
- \hat{P} - number of cell for which the value of membership function is equal to 1

The parameters α and β in equation (15) indicate how fast the vehicle's position was blurred or sharpened. Their values are entered separately for dilation ($\alpha, \beta < 1$) and erosion functions ($\alpha, \beta > 1$).

Another reason that the above two function have been introduced is to enable calibration of the model. Functions parameters have been established by experiment as: $\alpha = 0.90, \beta = 0.95$ for dilation and $\alpha = 3, \beta = 2$ for erosion. The vehicles' number generator has been implemented in the model.

In Fig. 4. the application window of the model implementation has been presented. The vehicles, which the position memberships function are expressed by the rhombus like shapes, move along the lanes (horizontal lines). On the upper line, the vehicles go from right to left side of the screen and on the bottom line – from left to right.

One can find out the vehicle's position, which is sharply defined at the beginning, is being increasingly blurred during its journey across the road section.

One special version of the automaton has been built to obtain the fundamental diagram. In this example the end of road section was connected to its beginning, to receive a loop. Then the road lanes have been randomly filled by the vehicles.

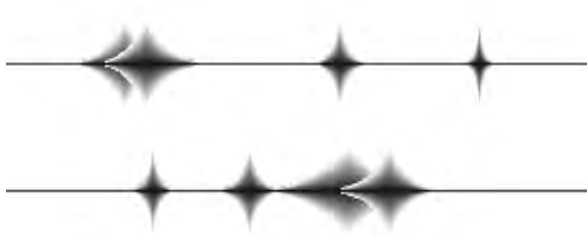


Fig. 4. Visualisation of the modeling process

The measurement of traffic flow has been made for several traffic densities (19 cases) and fixed flow at the opposite direction. The simulation result was introduced in Fig 5.

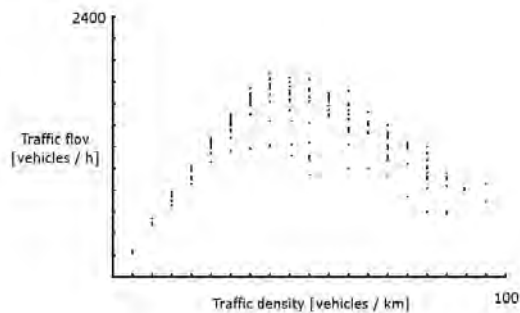


Fig. 5. The simulation traffic density diagram

4 Conclusions

The introduced description concerns some works undertaken for the traffic control processes by means of fuzzy cellular automaton applications efficiency analysis. The considered model and calculations allow us to take into account the uncertainty of the vehicles' position (placement on the traffic lane).

By representing all the variables describing the state of the vehicle's placement, using the fuzzy numbers, the random factor of the Nagel-Schreckenberg model was rejected successfully.

The facts that the traditional traffic control model contains probability factors, of the deceleration, makes it necessary considered vast amount of the data assigning the probability factors distribution. The introduced fuzzy model very encouraging results were obtained. The membership function of fuzzy numbers can be easily interpreted as the probability distributions. What is more the sufficient results are obtained after a single simulation run.

Taking the overtaking maneuver into consideration provides us with the traffic model closer to a traffic conditions, on the road stretch.

Extending the simulation procedures by two directions traffic, on neighboring lanes, allows us to carry out the overtaking maneuvers within the real traffic conditions.

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