Verhulst logistic map in the study of nonlinear and chaotic systems

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Abstract. *This article shows the basic tenets of the deterministic chaos theory that brings a new methodological framework and tools for exploring and understanding complex behavior in dynamical systems. We put an emphasis on description of the Verhulst logistic map which is one of the possible models and methods for researching dynamical systems that could develop to chaotic. In this paper we decribe the application of Verhulst method in biology, medicine, economy, political and social science, physics, and demography and information science.*

In the field of information science concepts of chaos theory proved as a valuable support in the research of interactions between information systems and their host organizations.The pioneer work with Logistic map was made in biology, but today this method is more frequently used in different scientific researces. It was discovered by the Belgian mathematician Pierre Francois Verhulst in the 1845 in order to describe the change in the population of a biological species.

Keywords: Verhulst map, science, chaos theory, nonlinear dynamics

1. Introduction

This article shows the basic tenets of the deterministic chaos theory that brings a new methodological framework and tools for exploring and understanding complex behavior in dynamical systems. We put an emphasis on description of the Verhulst logistic map which is one of the possible models and methods for researching dynamical systems that could develop to chaotic.

Also, we presented and described the possible applications of this theory in various fields of science, including information science. In the field of information science concepts of chaos theory proved as a valuable support in the research of interactions between information systems and their host organizations.The pioneer work with Logistic map was made in biology, but today this method is more frequently used in different scientific researces. It was discovered by the Belgian mathematician Pierre Francois Verhulst in the 1845 in order to describe the change in the population of a biological species. The map was used in the research paper in 1976 by the biologist Robert May, in part as a discrete-time demographic model analogous to the logistic equation created by Verhulst. Mathematically, the logistic map/equation is written

$$X_{n+1=} rx_n (1-X_n) \qquad (1)$$

where:

X_n is a number between zero and one, and represents the ratio of existing population to the maximum possible population at year n, and hence x_0 represents the initial ratio of population to max. population at year 0. *R* is a positive number, also known as control parametar and represents a combined rate for reproduction and starvation, and also represents the relationship between appearance we research and the enviroment of the appearance. Control parametar also shows us when and under what circumstances the appearance becomes chaotic.

This method could be also used for analysis and prediction of future development in different scientific fields with data from the past.

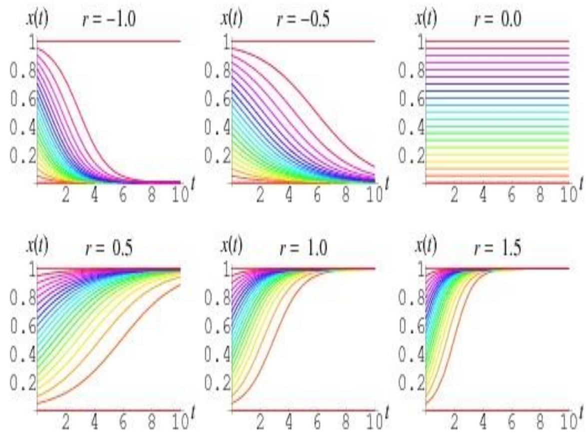

Figure 1. Different results of development of dynamic system with the same initial conditions and the change of control parametars. (Source:Wolfram Mathworld, http://mathworld.wolfram.com)

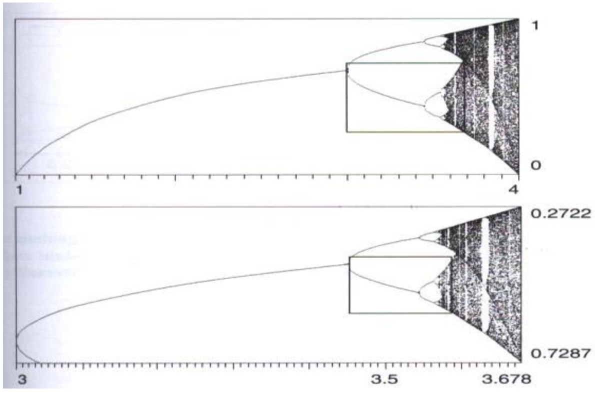

Figure 2. Bifurcation diagrams show self-similarity (on the horizontal axis are the values of the control parameter, and the vertical axis of development of the phenomena which we follow in the interval (0,1) The parameter value 3.5 opens the road to chaos. Source: [10]

2. Description of the Verhulst logistic map

Verhulst map is mostly used for monitoring the growth of an animal population and latter a number of economic, physical, medical, or other categories. Logistic equation is determined by a dynamic system that can become chaotic. Xn is a variable that describes the state of the dynamic system in the ***n***-th year. Dynamic and nonlinear system can be measured in the range between 0 and 1 which is derived from the condition that 0 < Xn <1, that number can not be less than 0 percent or more than 100 percent. In the case of Verhulst study, Xn denotes the number of insects (parents) who lived in the n-th year. Specifically, Xn is the ratio of the number of insects and the maximum number of insect N in the *n*-th year that could survive in this area, taking into account the amount of available food, etc. Interval (0,1) is particularly interesting for nonlinear systems because, while the number Xn increases, the part of the equation 1-Xn falls. This can be explained as follows; when numbers of insects or some other populations increase, it reduce the amount of food, which leads to a gradual decrease in population. Depending on the abruptness of ups and downs of the population, the speed and direction of development (growth or extinction) of the system will be determined. R is a control parameter, shows the influence of the environment on the phenomena under constant watch and its size is between 0 and 4. Size 0 indicates a weak interaction with the environment or extinction phenomena, while 4 indicates the chaotic and unpredictable behavior. The value of the control parameter depends of the living conditions, food availability, migration, enemies, predators, etc.

3. Application of the Verhulst/ logistic map/method

The analysis of events, we monitoring in the time series, could show that enviromental influence in a particular period had a connection with the events (for example. growth / decline of the population). After we detect those conections that happend in the past, we can also create scenarios of future developments. The basis of this procedure is comparing data (historic scenarios), obtained by calculations of the population equation (with changing initial conditions or the control parameters, giving you a variety of possible scenarios of development of population), with actual (observed) data. Than we can associate control parameters with the event that accompanies it, and that's the starting point for the prediction. After that comparation, we can continue with he future prediction using data from the past opservation. The scenario method consists of creating the future and describing ways to get out of the present reach of the future. The primary purpose of scenario analysis is to create a holistic and integrative picture of the future that can happen. In carrying out research projects of this type are almost always used graphic analysis, where the number of spicies or other categories are monitored continuously for a long time.

3.1.Research of chaos in biology, ecology and medicine

The knowledge that chaos could be detected in biology, especially in ecology, in the 70s of the 20th century resulted with substantial changes in these sciences. James Yorke and Robert May were the first scientist who take that knowledge in consideration [5]. In ecology, the animal population began to be seen as a dynamic system. Scientists used the basic tools of physics to describe the living world in which there are tens of millions of interconnected species.

James Yorke worked as a mathematician at the Interdisciplinary Institute of Physics and Technology, University of Maryland. He realized that in the works of Lorenz[1] and Smale there are messages that scientists, especially physicists, did not receive and have learned not to notice the chaos. Lorenz's experience of sensitive dependence on initial conditions, which in everyday life is everywhere, for some scientists were foreign. Yorke has realized that there is a mess, but that mathematicians and physicists want to discover regularities. In the past, scientists have seen chaotic behavior in numerous circumstances, but he was trying to correct and explain the mistake. [5] To demonstrate the behavior of populations with varying degrees of fertility, instead of individual diagrams, May and other scientists used the "bifurcation diagrams" to collect all information into a single image. The literature cites a diagram that shows how to change a parameter, in this case, population growth and the extinction of wildlife, fundamentally changing the behavior of this simple system. Parameter values are shown from left to right, and the final population is monitored on the vertical axis. Increase parameter values mean stronger stimulation system, but with increasing nonlinearity in it. Where the parameter is small (left), the population dies out. As the parameter increases (center), increasing the level of equilibrium. [10] But, with a further increase in the equilibrium and polarizes the population begins bifurcate between two different levels. (Figure 2) Then the system becomes chaotic (right) and the population passes through infinitely many different values. To explain, in the midst of this complexity suddenly appearing stable cycles. Although growth parameters, which means that non-linearity system starts getting stronger, suddenly appears a window with the correct period, such an odd period of three to seven, and the pattern of changes in the population is repeated in the three-year or seven-year cycle. Then start again doubling bifurcations in the following cycles of 3, 6, 12 .. or 7, 14, 28 .. and then once again turn into a chaotic area, as can be clearly displayed on the bifurcation diagram. [10] (Figure 2.) „Methods of nonlinear dynamics more frequently used in the detection of complex processes in the work of the heart and brain. The analysis of non-stationary time series of ECG and EEG investigate dynamic properties of the system. The problem of the presence of deterministic chaos in the work of the heart is now very active area of research, especially because it has been observed that some heart disease are changing the dynamics in a specific way, which can be detected only by nonlinear analysis.“ [15]

[1] Eduard Norton Lorenz was an American mathematician and meteorologist, and a pioneer of chaos theory. He discovered the strange attractor notion and coined the term *butterfly effect*.

3.2. Research of chaos in physics and technical sciences

In the middle and late 70's of 20th century, a number of young researchers showed great interest in chaos theory. At the University of California, Santa Cruz new campus gathered a group of talented students, graduates and later doctoral students, who described himself as a troupe of dynamic systems, while others have called Kabbalah chaos. They were Robert Stetson Shaw, Doyne Farmer, Norman Packard and James Crutchfield [5], gathered by the same thought that determinism can exist, but not quite. The challenge was their idea that deterministic systems can generate randomness. They did not even have an idea of what can cause nonlinearity, such as that one equation can bounce around randomly. They were thrilled with questions about determinism, nature of intelligence and direction of biological evolution. Physicist Albert Liebhaber also discovered chaotic behaviour in the systems he had researched. He performed the experiments on vibration. Vibrations often disturb nonlinear flow and transfer it from one type of behavior to another. He saw that the nonlinearity can stabilize the system, but also destabilize it and that nonlinear feedback controlled motion. „Today, the application of chaos theory in physics and technical sciences also explores the injection molding of thermoplastics.“ [13]

3.3. Research of chaos in economics and social sciences

„There are many companies and corporations today on the market, which is confirmed by regularity, but also there are many examples of incorrect decisions of their management, exposed to ruthless struggle for survival, almost like that in nature. Fighting for survival, the phases of the lifecycle are characteristics of living organisms. In organisms are very difficult to determine the causes and consequences of their actions. They behave almost chaotic, and chaos can only be established by means of probabilities, the approximate value of the so-called. fuzzy process. The interaction between an organization and its environment, is not predictable. Chaos theory explains many natural phenomena and found its application in many areas of human endeavor. The application of this theory has brought many new in explaining the behavior of business organizations in terms of eddy environment, and their transitions from a state of instability in the state of stability." [13]

According to Bhaskar[2], the paradigma of deterministic chaos is suitable for use in research in the field of social and political sciences. For this purpose, the different models and research strategies that work in specific levels of ontological reality in institutional structure. In experimental practice ideal models are often used in comparative economics and sociology, and historical models are mainly used in ideographic description the timing of certain specific events. [7] Aleksandar Halmi in 2002. explains that knowledge of mathematics of chaotic systems enables understanding of complexity theory in social phenomena. Without knowledge of complex systems of nonlinear structural equation is not possible to identify or interpret the chaotic behavior of a political system. It is impossible to identify and extract the chaotic behavior of some other manifest behaviors similar to chaos. Quantification of chaos in social research is one of the most important prerequisites for designing accurate conclusions in the studies. For such researches The Verhulst map is one oft he most propriate method. First example of studying chaotic effects in macroeconomics are in a paper by D. Gale published in 1973. year, and quoted him FI Murray and I. Stengos in 1988th year [9]. In this paper he analyzed the fluctuations in business cycles. Followed by several papers, such as for example the analysis by which J. Benhabib and Day in 1981st demonstrated the impact of chaos in changees the current consumption and the work in which JM Grandmond in 1985. analyzed the impact of chaos on incomes. In this series of work includes research which B. Berry 1991st [9] examined the longitudinal cycles of 1790th up in 1990. year, revealing that the alternating, namely periodic and cyclical macroeconomic trends, effects and cause more chaotic patterns. The author, along with macroeconomic variables, which are always heavily influenced by the transition from one stage of the cycle to another, points to various other influences that affect and cause changes in the economic sphere. [9]

On the securities market chaos theory has particularly extensive application. This market has all the properties of nonlinear dynamical systems. The effect of flapping wings quickly and strongly affects the global securities market, so for example even small changes in stock indexes in Tokyo, almost simultaneously causes changes of the stock market index in New York.

3.4. Research of chaos in demography

Verhulst logistic map is more appropriate to describe the animal than human populations. It is appropriate to describe a simple animal communities in which the propagation speed is proportional to the population size, and the relationships between certain types of relationships are predator and prey. Author Sergej Kapica [14] explains that in such systems, the maximum number of units is limited by the amount of natural food (prey). In certain circumstances, in such populations appear oscillatory patterns, which are observed in their natural habitats. On the other hand, these models are not suitable for the descriptions of the human population, because the circumstances of the growth and development of humanity quite different. For example, the growth of the human population is proportional to the square of the total number of individuals in the population and the human population is limited only by human inborn tendency reproduction . But some of the other human populations can be successfully modeled by logistic equations - one example of life and fertility. Logistic map, like any chaotic system, is extremely sensitive to initial conditions. Thus, the difference in the input data of a few parts per thousand after several iterative steps show different results.

3.5. Research of chaos in information systems

In information sciences, methods derived from chaos theory, could be successfully applied in the researching of the information flow in the organizations. For instance, we can describe an organization and it`s information system with a systems of nonlinear equations. By solving that systems of equations, we can predict a probability of organization development. „In this case, we can define an entropy, which is a measure of dezorganization. Opossite of that, the informations are the measure of organization." [19]

[2] Roy Bhaskar (born May 15, 1944) is a British philosopher, best known as the initiator of the philosophical movement of Critical Realism.

In treating organizations and the information systems that reside in them as non-linear systems,a number of assumptions are made [18]. McBride N. [16] highlights that change in the system is taken as being constant, and any apparent stable state is treated as temporary. „Organizations and their information systems cannot be decomposed into simple elements because the complex interactions between processes give rise to new emergent behaviour. System elements are interdependent and interactions between them are non-linear such that linear causal links cannot be made. Most significantly, for an interpretive use of chaos theory, effects within non-linear systems are non-proportional. Small inputs can have large effects, and large inputs result in no significant change.“ [16]

In the research paper McBride N. [16] said that the chaos theory could be used as a framework to draw out significant events and patterns of behaviour. „In particular, initial choice and conditions would affect the outcome of the system we explore. Repeating patterns of behaviour could be identified and examples of chaos theory concepts listed. Using concepts from chaos theory helped to explore significant issues arising from the explored system and helped in making sense of the progression of events over time. Application of chaos theory provided practical insights concerning the management of information systems strategies, but it may not result in new theory without further work.“ [16]

4. Conclusion

The most important terms in chaos theory are nonlinear dynamics or the dynamics of chaos. Nonlinear dynamics is a discipline studied nonlinearity, and chaos is just one of the behavioral shapes in nonlinear systems. Regardless of the nature of system, chaotic dynamic allways shows the same universal properties. There are many examples of nonlinear systems which are somewhat predictible, but in real life many systems in certain conditions cease to be predictable so the points within system started to move by chaotic patterns. The best way to explain this is to look into a bifurcation diagram which shows iteration of Verhulst population nonlinear equation (by changing control parametar values, that define environmental impact on the appearance). This is one of the commonly used method in researching chaotic behaviour. In this paper we showed that we can apply Verhulst logistic map in the research of chaotic behavior in differnt systems in different scientific fields and that method is especialy interesting in the research of chaos in information systems. Chaos theory brings many scientific disciplins close together and started to solve scientific problems by rejecting the stereotyped methods in science. Chaotic behaviours represented by curves that are depended on initial conditions, for the firs time was detected by amarican metheorologist Edward Lorenz, 50 years ago. He is considered as a father of the chaos theory and creator of frase “butterfly effect”. It is a phenomena based on conclusion that just one wingstroke of a butterfly in for example Brasil could couses a huricane in Asia. [7] This scientific field is open for more researces in the future.

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