Regional Pre-trip Sales Force Routing Derived From Multi-source Weber Problem

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Abstract. *This paper presents a case study on organization of pre-trip route planning and clustering for sales activity. Pre-trip route planning is a standard service of intelligent transportation systems (ITS). The study observed the territory of the Republic of Croatia which is represented with historical data for* 16 515 *points of sales. The solution should discover the number of sales regions, number of salesmen per region and planned routes for a single month. Constraints and input data are derived from real life points of sales data like sales potentials, geographic position and maximum number of customers for each route.*

Keywords. Optimization, routing, TSP, ITS, Regional clustering

1 Introduction

Launch of a new fast-moving consumer goods product on a market requires careful planning and distribution of sales forces which depends on sales potential in a particular point of sales. In order to minimize salesman travel expenses, the following variables are taken into consideration: the number of sales regions, number of salesmen per region and routes planned for a period of one month. In order to minimize cost, we wish to reduce the number of salesmen and their travel expenses. One of the focuses of this paper is how to ponder sales potentials in such a way that clustering points of sales gives priority according to the potential of sale and built clusters with centres which are placed as near as possible to the majority of consumers. Along with costs and priorities we will focus on the organization aspect of sales region as well. The type of marketing action is not considered in this case study.

The main contribution of this paper is a developed solution for the organization of pre-trip route planning which uses classical methods for solving Multi-source Weber problem, and (Traveling Salesman Problem) TSP together. The clustering points of sales in a sales region are determined by solving the Multi-source Weber Problem. We had to approach the problem in this way because standard geometric clustering methods like "centre of gravity" should not be applied because every point of sales has its own sales potential while geometric clustering does not take into consideration this additional ponder. Next phase is clustering inside every sale region and balancing the number of points inside each cluster. In the end, TSP route optimization on every cluster is performed.

The data used in planning is historical data on the observed territory of Croatia. The data includes geographical positions of 16 515 points of sales. Points of sale are mainly in retail businesses like groceries, drugstores, kiosks and convenience stores, department stores, shopping malls, retail chains, hypermarkets, supermarkets etc. Sales potential is calculated while taking into account the last two years of sales turnover and weight and quantity of delivered goods. As we had not focused on sales potential calculus, we suggest involving an expert for particular product in order to obtain even better results. In the ITS functional area Traveler information there are three main groups of information: Static and dynamic information about traffic network, Pre-trip and trip information and Support services to collecting, storing and managing information for other transportation planning activities (Business, Tourism, Entertainment, etc) [1]. Pre-trip information enables users to obtain useful information on the available modes, route, estimated travel times and others. This application gives the possibility to optimize various sales activities.

This paper is organized as follows. In the second section the Multi-source Weber's problem is discussed and the mathematical model is given. The nearest centre reclassification algorithm with Weiszfeld procedure is explained and used for clustering the regions and dividing regions in smaller clusters. Clustering regions in the Republic of Croatia on global level is conducted in section 2.2. In section 2.3 the new LS exchange heuristic is proposed for balancing clusters in a region. The whole solution for organization pre-trip route planning is explained in section 3. The results of the case study are in the section 4. The conclusion is made in section 5.

2 Clustering

The first aim of clustering is to form regions in Croatia on a global level. The second aim is the clustering inside regions and balancing the same number of customers in each cluster. First clustering should take into consideration the sales potential while the second clustering should take into consideration the maximum number of clients on every salesman route. The second clustering inside a region needs additional cluster balancing, which is described in section 2.3, after the application of the Multi-source Weber's problem on each region.

2.1 Multi-source Weber's problem

The location-allocation problem consists of locating centers for supplying or serving a destination and allocating, in an optimum fashion, a set of destinations (customers) that have a fix number of known locations and demands. Good examples for location problems are location of plants, warehouses, distribution centres where facilities provide a homogeneous service for sets of customers. The general problem may be stated as follows: the location, transportation costs and demands of each destination are known; the number, location and capacity of each source have to be determined [2].

Mathematical model in the continuous version of this location-allocation problem is known as a Multi-source Weber problem [3]. The objective is to generate m new centres in $\mathbb{R}^2$ which serve the demands of n destinations (fixed points) in such manner as to minimize the total transportation (or service) costs [4]

$$\min_{W,X} = \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} \parallel x_j - a_j \parallel \tag{1}$$

$$s.t. \sum_{J=1}^{m} w_{ij} = w_i, i = 1, ..., n \tag{2}$$

$$w_{ij} \geq 0, \forall_{i,j} \tag{3}$$

where $a_i = (a_{i1}, a_{i2})$ is the known location of customer i where $i = 1, ..., n$. $X = (x_{1,\dots}, x_m)$ denotes the matrix of location decision variables, with $x_j = (x_{j1}, x_{j2})$ being the unknown location of facility j where $j = 1, ..., m$. w_i is the given total demand or flow required by customer i where $i = 1, ..., n$. $W = (w_{ij})$ denotes the vector of allocation decisions variables, where w_{ij} gives the flow to customer i from facility j where $i = 1, ..., n$ and $j = 1, ..., m$. $\parallel x_j - a_i \parallel = \sqrt{(x_{j1} - a_{i1})^2 + (x_{j2} - a_{i2})^2}$ is the Euclidean norm [5].

2.1.1 The Nearest Center Reclassification Algorithm

The main difficulty in solving Multi-source Weber problem arises from the fact that the objective function is non-convex and, in general, contains a large number of local minima [3]. Among the bests heuristic methods for solving this problem is the Cooper's Locate-allocate procedure [5] which is based on Nearest Centre Reclassification Algorithm (NCRA) with Weiszfeld method [2]. NCRA is an iterative method more widely known under the name Nearest Mean Reclassification. According to [7] NCRA can be described as follows. Given a set of points $\mathcal{A} = \{a_1, a_{2,\dots}, a_n\}$, and a positive integer m, it is required to partition $\mathcal{A}$ into m non empty clusters $\Omega_{1,\dots}, \Omega_m$, so that each cluster Ω_i consists of the those points nearer to its center than to the centers of the other clusters $\Omega_k, k \neq i$. The initial partition Ω^0 is selected randomly. The $k - th$ iteration begins with a partition (or clustering).

$$\Omega^k = \{\Omega_1^k, ..., \Omega_m^k\} \tag{4}$$

The center x_i^k each Ω_i^k is computed (Algorithm 1, step 5), and points $a_j \in \Omega_i^k$ are reassigned to other cluster if closer to their centers than to x_i, (Algorithm 1, step 13). The algorithm stops (if no reassignments are possible) or proceeds with the new partition $\Omega^{k+1} = \{\Omega_1^{k+1}, \dots, \Omega_m^{k+1}\}$ reflecting the reassignments.

The general iteration is described as follows in [7]:

Algorithm 1 NCRA iteration k

1: Given a partition $\Omega^k = \{\Omega_{1,\dots}^k, \Omega_m^k\}$ of the set $a_{1,\dots}, a_n$
2: $r := 0$
3: **repeat**
4:     **for** $i = 1, ..., m$ **do**
5:         calculate the center x_i^k of Ω_i^k
6:     **end for**
7:     **for** $j = 1, ..., n$ **do**
8:         distances $d_{ij} = \parallel a_j - x_i^k \parallel, i = 1, ..., m$
9:     **end for**
10:     **for** $j = 1, ..., n$ **do**
11:         **if** $a_j \in \Omega_p^k$ and $d_{jl} = \min_{i=1,\dots,m} d_{ji} < d_{jp}$ **then**
12:             $\Omega_l^k := \Omega_l^k \cup a_j$
13:             $\Omega_p^k := \Omega_l^k \setminus a_j$(reassign a_j)
14:             $r := r + 1$
15:         **end if**
16:     **end for**
17:     $\Omega^{k+1} := \Omega^k$
18:     $k := k + 1$
19: **until** $r! = 0$

In this description of k iteration on NCRA one should be aware that reassignment in step 13 may leave a cluster Ω_p^k empty, having "lost" all its customers to nearest center. The next partition Ω^{k+1} may therefore have fewer than m clusters.

2.1.2 Weiszfeld procedure

The Weiszfeld procedure solves the problem of finding optimal location (x, y) of the cluster centre (Algorithm 1, step 5). It is an iterative procedure for the solution of a fixed point problem. An equation of the type $x = f(x)$ is solved iteratively generating a sequence $x^{(r)}$ where $x^{(0)}$ is a starting point and $x^{(r+1)} = f(x^{(r)})$ A starting point $(x^{(0)}y^{(0)})$ is weighted mean coordinates [2] (if the starting point or any iteration lands on known location of customers a_{i1}, a_{i2}, point which is not optimal, it should be adjusted a little to avoid dividing by zero). The Euclidean distance between customer i and center of the cluster in the iteration r is denoted as $d_i(x^{(r)}, y^{(r)})$.

$$x^{(r+1)} = \frac{\sum_{i=1}^{n} \frac{w_i a_{i1}}{d_i(x^{(r)},y^{(r)})}}{\sum_{i=1}^{n} \frac{w_i}{d_i(x^{(r)},y^{(r)})}} \tag{5}$$

$$y^{(r+1)} = \frac{\sum_{i=1}^{n} \frac{w_i a_{i2}}{d_i(x^{(r)},y^{(r)})}}{\sum_{i=1}^{n} \frac{w_i}{d_i(x^{(r)},y^{(r)})}} \tag{6}$$

The algorithm terminates when the distance between two consecutive iteration points is less than a given tolerance [6].

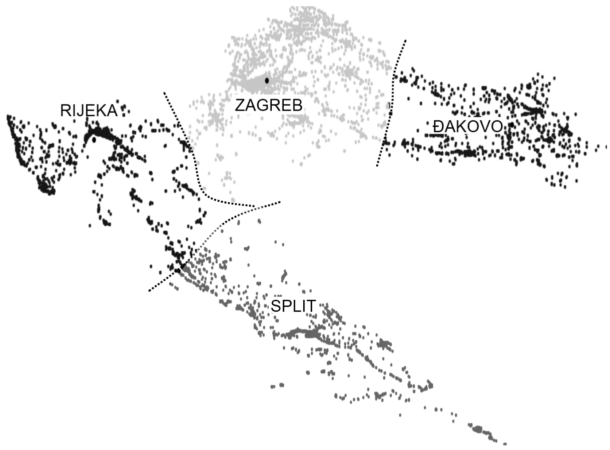

Figure 1: Regions with corresponding centers

Table 1: Global region

Region	Center	No. of customers
Primorje and Istra	Rijeka	3 805
Dalmacija	Split	3 154
Slavonija	Đakovo	2 492
Central Croatia	Zagreb	7 064
		$\sum$ 16 515

2.2 Clustering regions in the Republic of Croatia on global level

Salesman routing for such a large group of 16 515 points of sales (where points of sales are customers for our salesmen) should begin with forming a region. Forming regional clusters is a direct implementation of Multi-source Weber's problem where points of sales are locations, sales potential is ponder of every location and the number of regions is a subject of choice. The implementation of Algorithm 1 and Weiszfeld procedure solve clustering regions on global level.

After applying different number of sources (regions), the four regions are chosen. The calculated centers really match three big towns in Croatia: Zagreb, Rijeka and Split. The fourth centre is placed in Đakovo, which is not the largest town in that region Figure 1. The reason for not overlapping the calculated centre with the largest town in the region (Osijek) can be found in position of consuming population which is not directly coupled with the one largest city which dominates the region like in the other three regions.

The distribution of 16 515 points of sales by regions is shown in Table 1.

2.3 Clustering inside region with balancing

In every region a certain number of salesmen should cover the region. Our consideration assumes that for each salesman there 30 customers should be allocated. The time horizon is 20 working days. Salesman can first contact and afterwards possibly visit the assigned points of sales. Route optimization for visiting allocated customers described in section 3 is an additional benefit. The number of allocated customers per salesman is a parameter which can be easily tuned for particular needs of particular implementation.

Clustering inside a region has been performed by Algorithm 1 for solving Multi-source Weber's problem inside every region. Because of different number of customers inside clusters, additional balancing of clusters has to be done. The Algorithm 2 tries to establish the same number of customers in each cluster where *Max Points* constant is maximal number of customers per cluster. In our calculation the 30 customers per cluster is used.

Algorithm 2 Balance Clusters

```
while not number of points in clusters < Max Points do
    select cluster c0 > Max Points
    select cluster c1 < Max Points
    select point t0 from cluster c0 which is cluster c1 nearest
    add point t0 in cluster c1
end while
```

After the initial balancing by Algorithm 2, Algorithm 3 is proposed for exchanging customers between every pair of clusters in order to achieve better fitting of customers in clusters. Distance of customer from its center is used as a criterion of fitting.

Algorithm 3 LS Exchange

while there are points t_0 and t_1 that can exchange clusters **do**
    select cluster c_0, select point t_0
    select cluster c_1, select point t_1
    if $d(t_0, c_0) + d(t_1, c_1) > d(t_0, c_1) + d(t_1, c_0)$ **then**
        exchange points t_0 i t_1
    end if
end while

3 Solution for organization pre-trip route planning

Three phase solution for organization of pre-trip route planning and clustering for sales activity is done by Algorithm 4. Step 1 is named Cooper-Weiszfeld() and combines Algorithm 1 (the iterative locate-allocate procedure) and Weiszfeld procedure described in section 2.1.2. The cluster balancing is done in step 2 (Algorithm 2) and step 3 (Algorithm 3).

Algorithm 4 Pre-trip routing

1: Cooper-Weiszfeld
2: BalanceClusters
3: LS Exchange
4: TSP

Final route optimization inside each cluster is done by TSP procedure Algorithm 5 where simple nearest neighbour heuristic and 2-opt operator [8] is used.

Algorithm 5 TSP

while not all clusters c calculated **do**
    calculate NNH
    improve route with $2-$opt operator
end while

4 Results

The proposed solution gives useful quantity results for described case study of sales action in Croatia. Resources needed for visiting all 16 515 points of sales in observed territory of Croatia is the crew of 30 salesmen. Every salesman has to serve 30 customers per day. The time horizon for such operation is 20 working days. The summary of operation is shown in Table 2. The routes for the region Slavonia is shown in Figure 2.

Table 2: Number of salesman in the regions and cumulative distances for every region

Region	$\sum$ **dist** $[km]$	**No. of salesmens**
Primorje and Istra	15 799	$7(127\ routes)$
Dalmacija	13 482	$6(106\ routes)$
Slavonija	8 966	$5(84\ routes)$
Central Croatia	20 132	$12(236\ routes)$
	$\sum 58\ 379$	$\sum 30(553\ routes)$

5 Conclusions

The resource planning is important step in the successful sales activity. The first suggestion in planning resources like number of salesmen, time and routes is

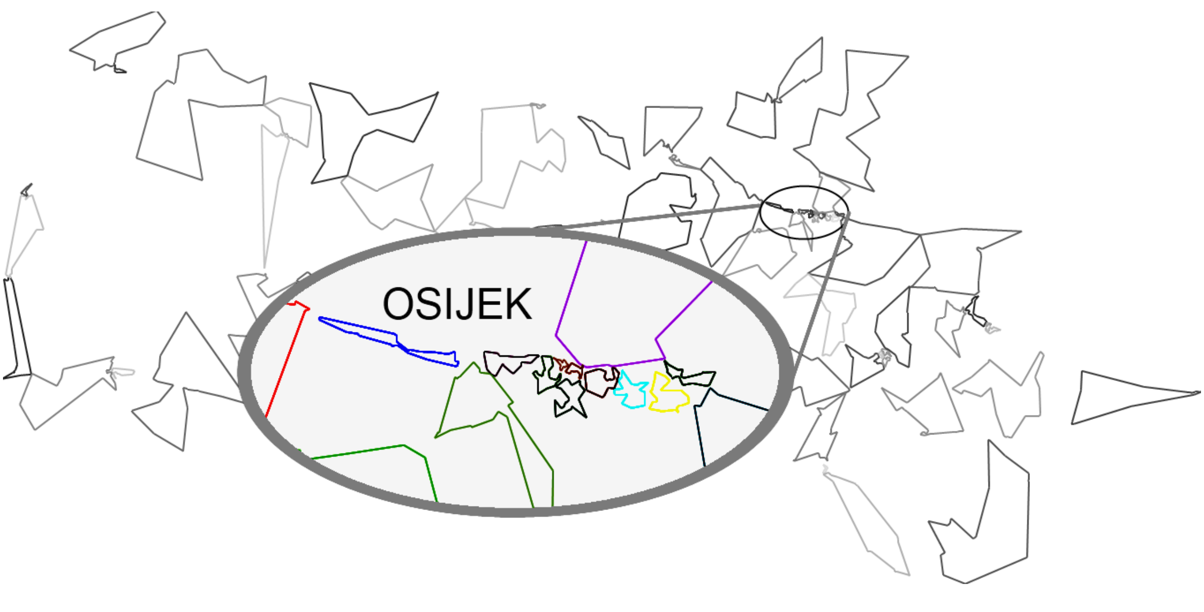

Figure 2: TSP solution for Slavonija region

regionalization. The region has to follow geographical distribution of consumers and the positions of large towns. Very often the administrative region is used. In this paper is clearly stated how to make regions with taking in the account the sale potential and geographical distribution of targeted points of sales.

Another clustering is suggested in order to divide regions into smaller clusters. We have used the same procedure for this location-allocation problem like in regionalization step before. The balancing for the same number of sales points in every cluster has been done afterwards. The balancing is done by described LS Exchange heuristic. The simple classical TSP route optimization is conducted on the end.

The ongoing developments which promise better results for regional pre-trip sales force routing follow the idea that after regionalization, one can apply the model and solutions for vehicle routing problem with time windows for better organizations of cluster inside region and route optimization.

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