# Dynamic capacity allocation of tollbooth plaza based on fuzzy logic

**Edouard Ivanjko, Marko Matulin, Štefica Mrvelj**
Faculty of Transport and Traffic Sciences
University of Zagreb
Vukelićeva 4, 10000 Zagreb, Croatia
{edouard.ivanjko, marko.matulin, mrveljs}@fpz.hr

**Abstract.** *This paper presents a fuzzy logic-based approach to control the number of active tollbooths in order to ensure desired Level of Service (LoS) with minimum of expenses. Input variables for proposed controller are: current queue length and current queue length change in two consecutive time steps. Controller output is the value that determines the type of change (increase, decrease or no change) of the active tollbooths number. A quality criterion for tollbooth plaza capacity control is proposed regarding the desired LoS defined with minimal and maximal waiting time constraints. Proposed criterion is used to evaluate the implemented fuzzy logic controller. For testing of the fuzzy logic controller, a stochastic queuing theory model is implemented as a Matlab/Simulink simulation model. Usability of implemented model and controller is also examined for system behavior prediction by using estimated values of traffic flow density. In order to provide relevant simulation results, real traffic flow data are used, including real toll payment time constants.*

**Keywords.** Tollbooth plaza, fuzzy logic controller, Markov process, dynamic capacity allocation.

## 1 Introduction

Road transportation systems have undergone a significant transformation in recent years. Emphasis is no longer solely on building roads to interconnect various destinations. Other factors such as real-time navigation, safe- and eco-driving, travel comfort, driver assistance, quick reaction to avoid incidents and optimal usage of road transport infrastructure are becoming more and more important. The importance of some of these factors is highlighted by researchers in [6] and authorities in [8]. Hence, to ensure efficient usage of the traffic system capacity, intelligence needs to be added to its control systems. These control systems enable an autonomous response of the traffic infrastructure to the changes of traffic flow or weather conditions, such as activating service lanes as full road lanes to increase maximal possible traffic flow, activating dynamic traffic signs to inform drivers about new road conditions, rerouting vehicles to avoid congestion or an incident situation, etc.

One of the elements that have a significant influence on road capacity and vehicle travel times are tollbooth plazas. They are a possible bottleneck and can reduce travel comfort, [5]. In order to reduce their negative impact on traffic flow, various systems have been developed, e.g. self service toll payment for drivers with credit cards, electronic toll collections systems based on RFID or video technology, etc. Such systems reduce time needed for toll collection; hence reducing or completely avoiding queueing in off-peak traffic hours, [6], [13].

When these payment systems are not available, the most common approach used for reduction of average waiting time at tollbooth plazas is to increase the tollbooth plaza capacity by building more tollbooths. Due to the known properties of traffic flow (its dynamics during the day, week, month or a whole year) it is important to allocate tollbooth plaza capacity efficiently. Efficient tollbooth plaza capacity allocation is crucial for roads with significant differences of traffic flows in peak and off-peak hours. This means finding the balance between the desired Level of Service (LoS) and the number of active tollbooths (i.e. costs of labor and other related costs) in any given moment. So tollbooth plaza capacity must be dynamically allocated and based on the incoming traffic flow; not just being managed with all tollbooths being active the whole day.

State-of-the-art in sensory technology and measuring equipment enables recording and calculation of various variables that are needed for tollbooth plaza queuing system analysis. Some of these variables are: incoming traffic flow volume, vehicle types, queue length, service time, etc. Furthermore, intelligent traffic signs and other intelligent transport system solutions enable quick information forwarding to drivers. For instance, reduction of maximum allowed speed on motorway will slow down the traffic flow, ergo incoming traffic load on a tollbooth plaza will be reduced as well.

Tollbooth plaza capacity can be determined manually, by observing queue lengths. When queue length exceeds its specific maximum (determined by tollbooth plaza management personnel), number of active tollbooths may be increased and vice versa. Obvious drawback of such method is that firstly delivered LoS must be significantly degraded in order to evoke a change of capacity, or in the other case, queue length has to drop significantly before the active tollbooths number can be reduced. In both cases capacity is not allocated efficiently. When applying this "manual" method another problem may emerge: there is a possibility that in a given moment there is not enough workers on the tollbooth plaza site and so not all available tollbooths can be active. In this case system response to dynamic changes is slower or even prolonged, which results in traffic congestion and unnecessary long waiting times. When using intelligent control systems for tollbooth plaza capacity allocation or prediction, this problems can be avoided or reduced. This paper proposes such a control system based on fuzzy logic.

Organization of this paper is as follows: section two explains the tollbooth plaza LoS problem; tollbooth plaza model used in this paper is described in section three; section four presents the proposed fuzzy logic controller; and in section five simulation results are presented and discussed. Paper ends with conclusions and future work ideas.

## 2 Tollbooth plaza LoS problem

Performance evaluation of toll facilities requires a good understanding of their unique characteristics and constraints, and the definition of the right measures of effectiveness (MOEs) that help explain the LoS perceived by users, [4]. The length of the queue and the time spent in the queue are the two most significant MOEs that reflect both how the toll collection system is performing and what users perceive as its LoS. In this paper time spent in the queue waiting for service, will be used as primary MOEs for evaluation of the proposed fuzzy logic controller.

LoS framework defined in [14] comprises of six LoS classes for signalized intersections with defined waiting time intervals given in Table 1. According to available literature [2], [5], [6], [9] these six classes of LoS may be used for tollbooth plaza analysis.

| LoS class | Waiting intervals |
|---|---|
| A | $\geq\ 0-10\ [s]$ |
| B | $\geq 10-20\ [s]$ |
| C | $\geq 20-35\ [s]$ |
| D | $\geq 35-55\ [s]$ |
| E | $\geq 55-80\ [s]$ |
| F | $\geq 80\ [s]$ |

Table 1: LoS classes

## 3 Tollbooth plaza model

As mentioned before, tollbooth plazas are an integral part of motorway systems on which users, i.e. drivers, have to pay the toll for a service which was provided to them (e.g. for the use of motorways, bridge crossings, tunnel passing, etc.). After analyzing the traffic process on motorways it can be concluded that tollbooth plazas introduce disruption into the normal traffic flow by slowing it down. In this case, accurate capacity allocation may decrease negative effects such as decrease of average driving speed ($\bar{v}$, measured in kilometers per hour), increase of average waiting time in the queues ($T_w$, measured in seconds) and increase of average queue length ($L_w$, measured in number of vehicles in queue in any given moment) with respect to minimal economic cost. Economic cost is in this case related to needed tollbooth personnel and other cost related to running individual tollbooths.

Like in most queuing systems, traffic process in tollbooth plazas have some general characteristics, as follows:

1. System capacity is determined with a number of tollbooths $m$ which are active in a given moment;
2. Each individual tollbooth serves only one user for a certain time period, i.e. average service time $T_s$ measured in seconds may be defined to describe this process;
3. With the knowledge of input traffic flow ($\varphi$) characteristics, intensity at which users arrive into the queuing system can be defined ($\lambda$, measured in vehicles per hour);
4. If there are no available free tollbooths, users form a queue (queue properties can be described with parameters $T_w$ and $L_w$).

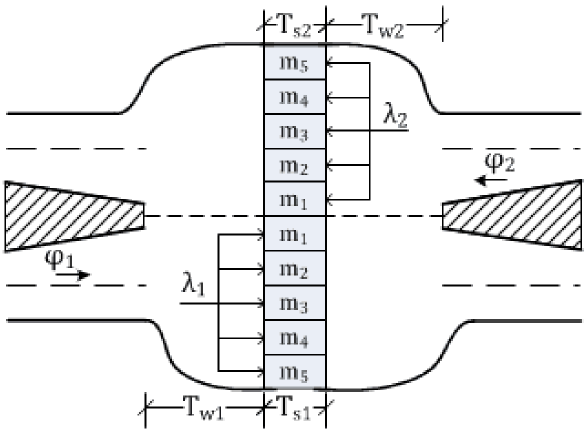


Figure 1: General two-direction tollbooth plaza

As it can be seen in Fig. 1, general two-direction tollbooth plaza consist of incoming lanes, queuing area, tollbooths and outgoing area. Tollbooths can be active or idle regarding toll collecting and can be assigned to one direction of traffic flow only. In general, certain number of tollbooths can be shared between traffic flow directions if traffic conditions in each direction allow that (e.g. if incoming traffic load in one direction is larger than incoming traffic load from the other direction).

Variable indexes in Fig. 1 denote one of the two directions of traffic flow. In this paper only one traffic flow direction at the tollbooth plaza is considered. Authors in [2] and [9] used a similar modeling approach, but only with fixed number of active tollbooths without the control part proposed in this paper. In total there are 10 tollbooths on the tollbooth plaza, five for each traffic flow direction. For simplicity reasons in our analysis we assume that on all 10 tollbooths classical payment method is applied.

In order to model one traffic flow direction of described tollbooth plaza a Markovian M/M/1 model for queuing systems is used. According to [1], [3], [11] model assumptions are:

1. Incoming traffic load is equally distributed to all tollbooths;
2. In a short time period traffic flow in the tollbooth plaza area can be described as stationary;
3. Vehicle inter-arrival times are distributed exponentially;
4. Service time is distributed exponentially;
5. There is enough space for all vehicles to get into the queue;
6. One tollbooth can serve only one vehicle at a time;
7. Service discipline is First Come First Service (FCFS);
8. Outgoing queue does not affect tollbooth service time.

Proposed tollbooth plaza model can be described mathematically using the following equations:

$$A = \lambda \cdot T_s, \tag{1}$$

$$\rho = \frac{A}{m}, \tag{2}$$

$$T_w = T_s \frac{\rho}{1-\rho}, \tag{3}$$

$$L_w = T_w \frac{\lambda}{m}, \tag{4}$$

where $A$ is traffic load (i.e. expected number of occupied tollbooths in any given moment) and $\rho$ is traffic load of a single tollbooth. Input variables to the tollbooth plaza model are traffic flow intensity $\lambda$, and number of active tollbooths $m$. Incoming traffic flow intensity depends on a particular traffic situation. The number of active tollbooths will be computed using the current number of active tollbooths and its change obtained from the fuzzy logic controller. Model output values are average waiting time in the queue $T_w$ and queue length $L_w$.

# 4 Fuzzy logic controller

Proposed fuzzy logic controller changes the number of active tollbooths dynamically, i.e. relative to incoming traffic load on the tollbooth plaza. This implies that the controller must have the information about current number of vehicles in queue on the tollbooth plaza and it has to have information about the maximum queue length which is allowed for a specific LoS class and for a specific number of active tollbooths. For example, using the data from Table 1, eqs. 1 to 4, and average $T_s$ value of 21 $s$, it can be calculated that with 4 active tollbooths LoS class D can be maintained if cumulative maximum queue length does not exceed 7.6 vehicles in average (1.9 in front of each tollbooth). Difference between current queue length $L_{w_t}$ and maximum queue length $L_{w_{LoS}}$, which is allowed for a specific class, is defined as the first input variable into proposed fuzzy controller:

$$L_{w_1} = L_{w_t} - L_{w_{LoS}}. \tag{5}$$

If the result of eq. 5 is a positive number, then current LoS is lower than predefined, and number of active tollbooths can be reduced. In opposite situation, if the result is a negative number, then current LoS is higher than predefined, and number of active tollbooths should be increased. Since value of $L_{w_1}$ can oscillate $\pm 1$ vehicle, and with such small deviations it would not be efficient to change tollbooth plaza capacity, it is necessary to define different fuzzy sets for $L_{w_1}$. Hence, five linguistic variables are defined: $+LD$ (large positive difference), $+SD$ (small positive difference), $ND$ (no difference), $-SD$ (small negative difference), and $-LD$ (large negative difference). For two sets ($+LD$ and $-LD$) trapezoidal membership functions were defined, and for other three sets ($-SD$, $ND$ and $+SD$) we used triangular membership functions.

Second input variable into proposed fuzzy logic controller ($L_{w_2}$) represents difference between current queue length in two consecutive 15-minute intervals:

$$L_{w_2} = L_{w_{1_t}} - L_{w_{1_{t-1}}}, \tag{6}$$

where $t$ and $t-1$ denote two consecutive 15-minute intervals, respectively. In order to make a gradation of the second input variable $L_{w_2}$, five fuzzy sets were defined: $LI$ (large increase), $SI$ (small increase), $NC$ (no change), $SD$ (small decrease), and $LD$ (large decrease).

Proposed fuzzy controller output variable represents the instruction to the tollbooth plaza management personnel whether to increase/decrease the number of active tollbooths or to leave it unchanged. Similarly to the two input variables, five instructions, representing five linguistic variables necessary to complete the inference process, were defined. Names and notations are equal to second input variable $L_{w_2}$, but with different meaning: number of active tollbooths change. This number can be changed as follows: (i) increase for 2, (ii) increase for 1, (iii) no change, (iv) decrease for 1, and (v) decrease for 2. With 2 input variables ($L_{w1}$ and $L_{w2}$) and 5 linguistic values for each input variable, in total 25 fuzzy rules can be defined.

After the examination of the incoming traffic flow dynamics and simulation testing, it can be concluded that smaller set of fuzzy rules (less than 10) can result with good tollbooth plaza capacity allocation. After series of tests and simulations under different settings and input values, for the aggregation system we have chosen disjunctive system of rules and mean max membership defuzzification method.

# 5 Simulation results

For the proposed control scheme evaluation appropriate model is implemented in Matlab/Simulink. Its block scheme is given in Fig. 2. As it can be seen, integral parts of the model are: fuzzy logic controller block which prepares the input signals, and the tollbooth-plaza model block. Tollbooths-plaza model has two inputs: the instruction about the change in the active tollbooths number, and incoming traffic flow intensity. This model derives different outputs, but most relevant for our research is the queue length. Queue length is the variable which is used in the proposed control scheme. Other outputs include signals needed for detailed analysis: cumulative number of vehicles in a queue, waiting time (cumulative and per active tollbooth), tollbooth plaza traffic load, and current number of active tollbooths.

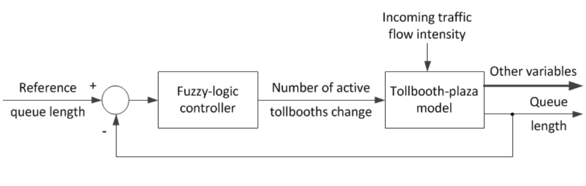


Figure 2: Tollbooth control block scheme

Proposed input to the control scheme is the reference queue length which is directly proportional to the average waiting time (see eq. 4). In our simulation we defined the D class as the one which we want to maintain, thus reference queue length per active tollbooth had to be set to 2.

For the first performance evaluation of proposed control scheme only one direction of traffic flow on the tollbooth plaza is analyzed. Parameters which were used are as follows:

- minimum number of active tollbooths was set to 2;
- maximal number of active tollbooths was set to 5;
- average service time was set to 21 seconds.

The value for average service time is obtained from measurements which were conducted on cash-payment tollbooth plazas in Croatia (for further reference see [10]). Finally, initial number of active tollbooths was set to 2.

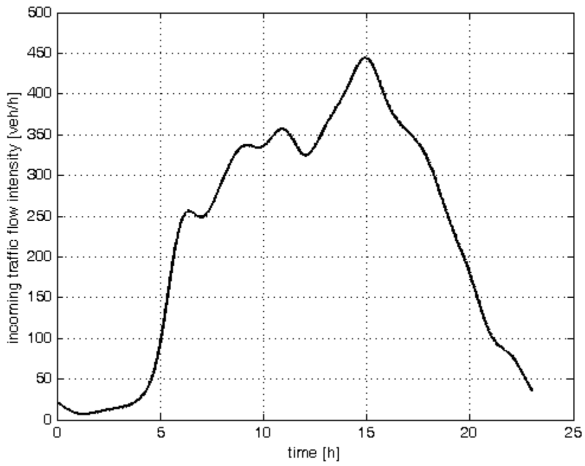


Figure 3: Typical incoming traffic flow intensity

To ensure relevant simulation results, real data sets were used to define incoming traffic flow intensity. The data was obtained from Slovenian Road Agency in digital format (CD:ISSN 1580-3864). The measurements were conducted using automatic counters which were installed in different parts of the Slovenian motorway network. To ensure representativeness of the input data we arbitrary selected one typical day in which high traffic flow variations exist (from the counter no. 188). This way we objectively tested the controller ability to react to the dynamic changes.

Obtained data records also contained the information about the time of the measurements and types of vehicles which passed through the measuring point. On each measuring point the data was recorded for two directions of travel. Traffic flow rate measurements are sampled in one hour intervals. In order to obtain the data about the incoming traffic flow appropriate for simulation, vehicle categories for one traffic flow direction were summed; missing data between hour intervals were interpolated using a spline polinom. Results of this interpolation are shown in Fig. 3. Slovenia is one of Croatia's neighboring countries and each year during the summer holiday seasons, motorized tourists

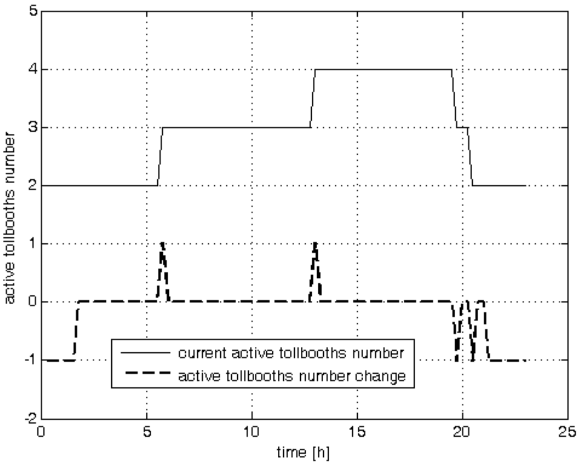


Figure 4: Number of active tollbooths change

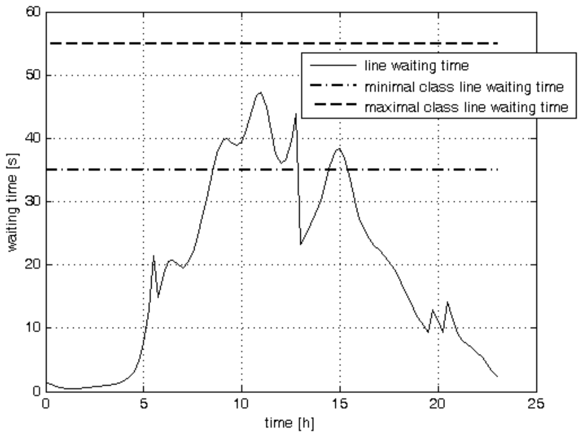


Figure 5: Waiting time analysis

traverse through Slovenia and come to Croatia. We therefore believe that the data obtained from Slovenian motorway can be used to evaluate the performance of our controller (as we mentioned before, $T_s$ value is collected from measurements of average service time on Croatian tollbooths).

Fig. 4 depicts the changes in the number of active tollbooths during one day. As it can be seen, maximum number of active tollbooths has not been reached. Furthermore, it is evident that current number of active tollbooths (full line) roughly matches the incoming traffic flow data from Fig. 3, which proves that controller reacts appropriately to the dynamic input. For example, in the midnight hours, when there is minimum traffic flow intensity, controller wants to decrease the number of active tollbooths (dashed line), but that is not possible due to the defined minimum number of active tollbooths which was set to 2. When traffic flow intensity increases, controller reacts and gives the instruction to increase the number of tollbooths. After the peak period of the day rather steep decrease of traffic flow intensity begins (Fig. 3). In this case fuzzy logic controller gives the instruction to decrease the number of active tollbooths. It is clear that in both cases (during traffic flow increase and decrease) fuzzy logic controller reacts appropriately, i.e. the controller wants to maintain the desired LoS class which was set to D class.

Parallel analysis of simulation results, which are presented in Fig. 4 and Fig. 5, shows that any increase of the number of active tollbooths causes a decrease in waiting time.

As mentioned above, to enable a quantitative evaluation of proposed fuzzy logic controller, LoS classes are used. Each class has its own waiting time interval. Since we set the desired LoS class to D, Fig. 5 shows the lower and upper $T_w$ boundaries of that class (dash-dotted line and dashed line, respectively). Based on these results, it can be concluded that fuzzy logic controller fulfilled its task. Specifically, with only $4$ changes of the number of active tollbooths (Fig. 4) the desired D class was maintained throughout entire simulation time ($24$ hours), despite the fact that the incoming traffic flow intensity changed significantly during the day. Waiting time results depicted in Fig. 5 show that LoS on the tollbooth plaza was in the D class or better during the simulation (full line).

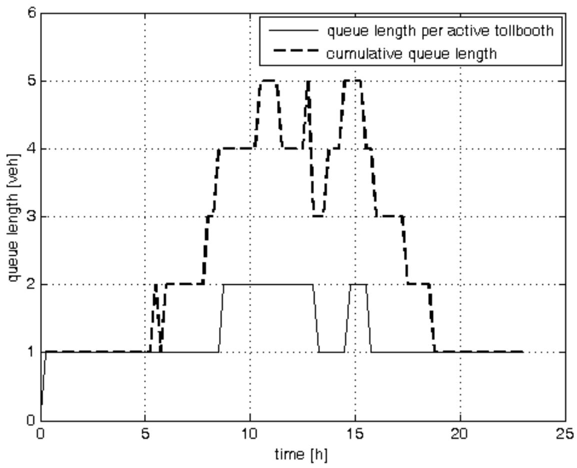


Figure 6: Number of vehicles in a queue

Apart from the $T_w$ evaluation parameter, another important parameter is the average number of vehicles in the queue, i.e. average queue length. Results of this analysis are presented in Fig. 6. Two sets of data are presented: cumulative number of vehicles in a queue (i.e. total number of vehicles on a tollbooth plaza waiting for a service) and the number of vehicles in front of each active tollbooth. These sets are presented with dashed and full line, respectively. In order to obtain the average values we approximated the output results to the nearest higher integer value.

As it was already mentioned, proposed tollbooth plaza was modeled as a Markovian $M/M/1$ queuing

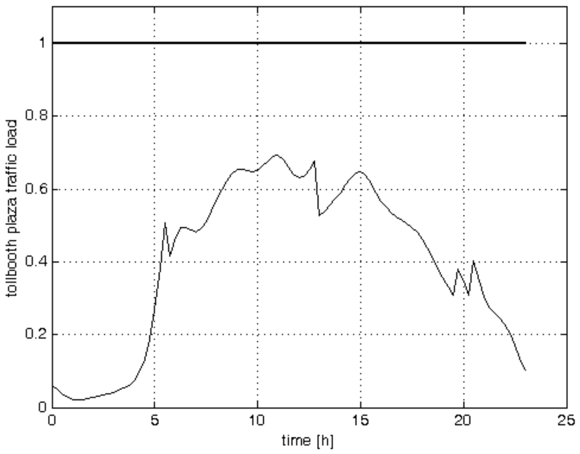


Figure 7: Tollbooth plaza traffic load

system model, hence average traffic load by each active tollbooth (server) can be calculated and analyzed. In our case, average traffic load per tollbooth did not exceed 0.7 or 70 % (Fig. 7), ergo it can be concluded that there is still available resources which could be used to respond to additional incoming traffic flow increase. More specifically, with the proposed fuzzy logic controller, D class could also be maintained with higher incoming traffic flows.

Based on the data of the incoming traffic flows it is possible to predict future traffic flows by using prediction systems (for further reference see [7]), ergo predictions can be made regarding the tollbooth capacity (i.e. man power and labor costs). Therefore, it is clear that the proposed fuzzy based controller can be useful when improving overall efficiency of tollbooth plazas and in predicting future resources.

# 6 Conclusion and future work

In this paper a fuzzy logic controller for dynamic capacity allocation of tollbooth plaza is proposed. For evaluation purposes a Markovian M/M/1 model was used to model the tollbooth plaza queuing system. Simulation model for Matlab/Simulink has been made including parameters obtained from real world measurements. Also, real traffic flow data measurements were used to obtain realistic simulation results.

Obtained results show that fuzzy logic controller can successfully maintain the desired LoS class by dynamically changing the tollbooth plaza capacity. Furthermore, we showed that with only 4 changes in the number of active tollbooths during one day, desired LoS class can be achieved and preserved (no tollbooths was occupied for only short period of time). This proves that the proposed fuzzy based controller efficiency allocated the capacity and helped to make solid predictions about needed resources. More importantly, the controller helps to ensure relatively stable LoS which is important for travel planing and general satisfaction of travelers.

In our future research we will include second direction of traffic flow in the analysis. This will allow us to investigate the tollbooth system behavior when traffic load per one tollbooth tends to maximum ($\approx 1$) or when it exceeds the maximum. In this case, tollbooths which were predefined to one direction of traffic flow can be re-assigned to other direction in order to decrease that high traffic load (i.e. certain number of tollbooths could be shared between different directions).

Additional analysis can be made when different toll collection systems are implemented, i.e. when different service times are present, depending on the applied technology.

Lastly, in our research, the process of re-entering the motorway after completing the payment was ignored. This process also represents classical queuing problem which can be modeled with Markovian models.

# 7 Acknowledgments

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